VERSION 1

MATH 113 - EXAM 1 - SOLUTIONS

1. (a) $\lim_{x \to 2} f(x) = 3$

(b) Yes (because $\lim_{x \to 0} f(x) = 1 = f(0)$)

(c) $\lim_{x \to 1} f(x)$ does not exist (because the one-sided limits differ)

(d) $\lim_{x \to 3} f(x) = 0$

(e) $\lim_{x \to -2^-} f(x) = +\infty$

(f) NO. $\lim_{x \to 2} f(x) = 3$ but $f(2) = 1$.

(g) f is continuous on

$[-4,-2) \cup (-2,1) \cup (1,2) \cup (2,3) \cup (3,4]$.

VERSION 1

2 (a) $$\lim_{x\to -1} \frac{x^2-x-2}{x^2-1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x-2)}{(x-1)\cancel{(x+1)}}$$

$$= \lim_{x\to -1} \frac{x-2}{x-1} = \frac{-3}{-2} = \frac{3}{2} \,//$$

(b) $$\lim_{t\to 0} \frac{\sqrt{t+4}-2}{t} = \lim_{t\to 0} \frac{\sqrt{t+4}-2}{t} \cdot \frac{\sqrt{t+4}+2}{\sqrt{t+4}+2}$$

$$= \lim_{t\to 0} \frac{(t+4)-4}{t(\sqrt{t+4}+2)} = \lim_{t\to 0} \frac{\cancel{t}}{\cancel{t}(\sqrt{t+4}+2)}$$

$$= \lim_{t\to 0} \frac{1}{\sqrt{t+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4} \,//$$

(c) $$\lim_{x\to 2} \frac{\frac{x+1}{x-1}-3}{x-2} = \lim_{x\to 2} \frac{1}{(x-2)}\left(\frac{(x+1)-3(x-1)}{x-1}\right)$$

$$= \lim_{x\to 2} \frac{1}{x-2}\left(\frac{-2x+4}{x-1}\right) = \lim_{x\to 2} \frac{1}{\cancel{x-2}}\left(\frac{-2\cancel{(x-2)}}{x-1}\right)$$

$$= \lim_{x\to 2} \frac{-2}{x-1} = -2 \,//$$

VERSION 1

3. (a) $\lim_{x \to \infty} \frac{x^2-3x}{x^2-4} = \lim_{x \to \infty} \frac{(x^2-3x)(1/x^2)}{(x^2-4)(1/x^2)}$

$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{3x}{x^2}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \lim_{x \to \infty} \frac{1-\frac{3}{x}}{1-\frac{4}{x^2}} = 1$ //

(b) $\lim_{x \to -2^+} \frac{x^2-3x}{x^2-4} = \lim_{x \to -2^+} \frac{x(x-3)}{(x-2)(x+2)}$

If $x > -2$ but close to -2, $x < 0$, $x-3 < 0$

$x-2 < 0$ and $x+2 > 0$, so $\frac{x(x-3)}{(x-2)(x+2)} \left[\frac{(-)(-)}{(-)(+)}\right] < 0$

Therefore $\lim_{x \to -2^+} \frac{x^2-3x}{x^2-4} = -\infty$ //

(c) $\lim_{x \to 3} \frac{x^2-3x}{x^2-4} = \frac{0}{5} = 0$ //

4. $|f(x)-3| < 1.5$ means $f(x) \in (1.5, 4.5)$. From the graph, this holds when $x \in (1.5, 2.5)$ which is $|x-2| < .5$. Therefore $\boxed{\delta = .5 \text{ works}}$

VERSION 2

MATH 113 – EXAM 1 – SOLUTIONS

1 (a) $\lim_{x \to 2} f(x) = 3$

(b) No (because $\lim_{x \to 2} f(x) = 3$ but $f(2) = 1$)

(c) $\lim_{x \to 1^+} f(x) = 0$

(d) $\lim_{x \to 3^-} f(x) = 0$

(e) $\lim_{x \to -2^+} f(x) = -\infty$

(f) Yes ($\lim_{x \to 0} f(x) = 1 = f(0)$).

(g) f is continuous on

$[-4,-2) \cup (-2,1) \cup (1,2) \cup (2,3) \cup (3,4]$

VERSION 2

2. (a) $\displaystyle \lim_{x\to 1} \frac{x^2+x-2}{x^2-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+1)}$

$\displaystyle = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{3}{2}$ //

(b) $\displaystyle \lim_{t\to 4} \frac{\sqrt{t+12}-4}{t-4} = \lim_{t\to 4} \frac{\sqrt{t+12}-4}{t-4} \cdot \frac{\sqrt{t+12}+4}{\sqrt{t+12}+4}$

$\displaystyle = \lim_{t\to 4} \frac{(t+12)-16}{(t-4)(\sqrt{t+12}+4)} = \lim_{t\to 4} \frac{\cancel{t-4}}{\cancel{(t-4)}(\sqrt{t+12}+4)}$

$\displaystyle = \lim_{t\to 4} \frac{1}{\sqrt{t+12}+4} = \frac{1}{\sqrt{4+12}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{8}$ //

(c) $\displaystyle \lim_{h\to 0} \frac{\frac{1}{5+h}-\frac{1}{5}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{5-(5+h)}{5(5+h)}\right)$

$\displaystyle = \lim_{h\to 0} \frac{1}{\cancel{h}}\left(\frac{-\cancel{h}}{5(5+h)}\right) = \lim_{h\to 0} \frac{-1}{5(5+h)} = \frac{-1}{25}$ //

VERSION 2

3. (a) $\displaystyle\lim_{x\to-\infty}\frac{x^2-3x}{x^2-4}=\lim_{x\to-\infty}\frac{\frac{x^2-3x}{x^2}}{\frac{x^2-4}{x^2}}$

$\displaystyle=\lim_{x\to-\infty}\frac{1-\frac{3}{x}}{1-\frac{4}{x^2}}=1$ //

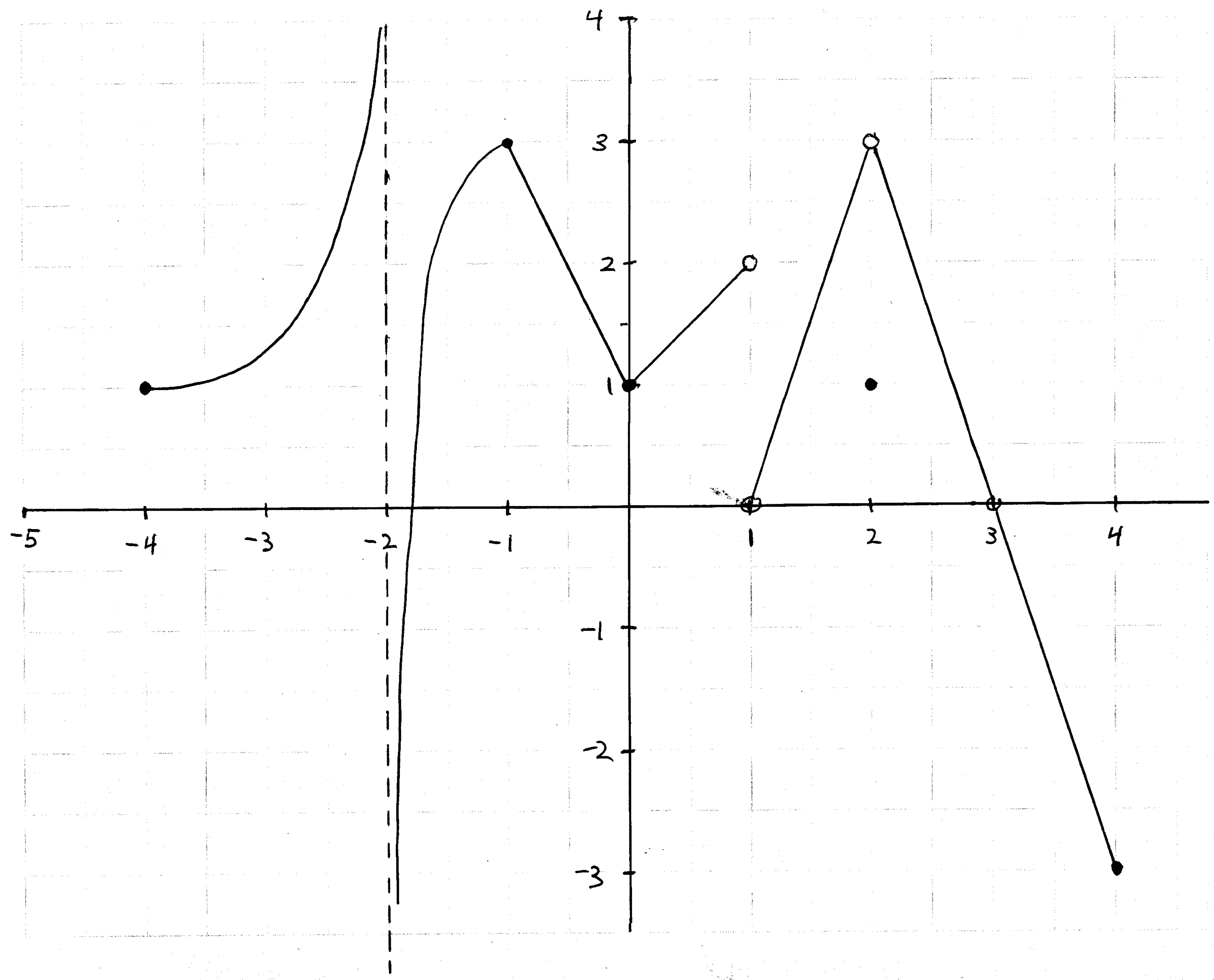
(b) $\displaystyle\lim_{x\to2^-}\frac{x^2-3x}{x^2-4}=\lim_{x\to2^-}\frac{x(x-3)}{(x-2)(x+2)}$

If $x<2$ but near 2, $x>0$, $x-3<0$, $x-2<0$, $x+2>0$ so $\displaystyle\frac{x(x-3)}{(x-2)(x+2)}\left[\frac{(+)(-)}{(-)(+)}\right]>0.$

$\displaystyle\therefore\lim_{x\to2^-}\frac{x^2-3x}{x^2-4}=+\infty$

(c) $\displaystyle\lim_{x\to0}\frac{x^2-3x}{x^2-4}=\frac{0}{-4}=0$

4. $|f(x)-3|<1.5$ means $f(x)\in(1.5,4.5)$
From the graph, this holds when $x\in(1.5,2.5)$
which is $|x-2|<.5$ Therefore $\boxed{\delta=.5 \text{ works}}$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