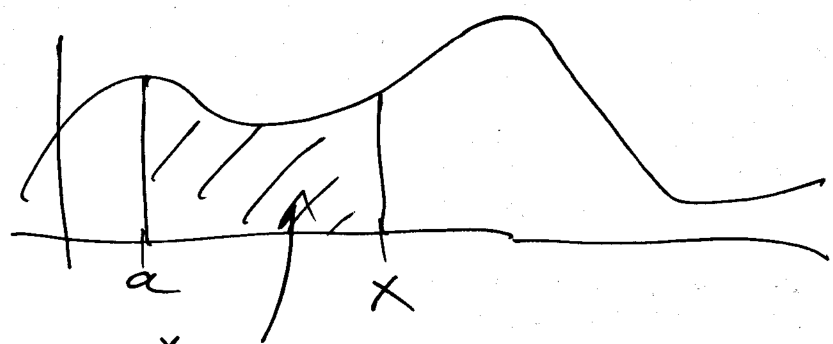
Fundamental Theorem of Calculus

(I) $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$



$\int_a^x f(t)\,dt$

(II) $\underbrace{\int_a^b f(x)\,dx}_{\text{net area under graph of } f(x)} = \underbrace{F(b)-F(a)}_{\text{total change in antiderivative of } f(x).}$ $\quad F'(x)=f(x)$

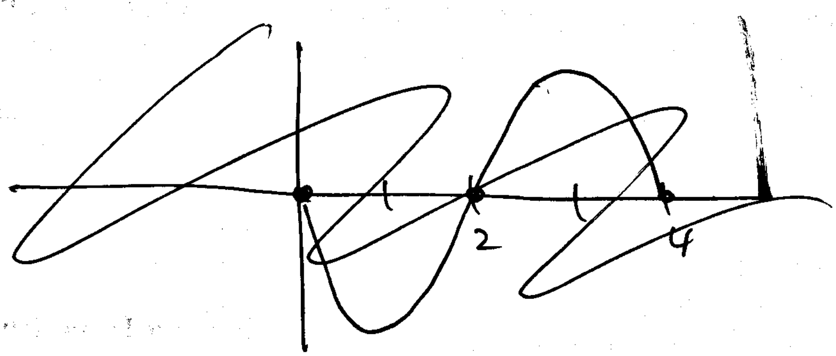
#34) $\int_0^4 x(x-2)(x-4)\,dx$

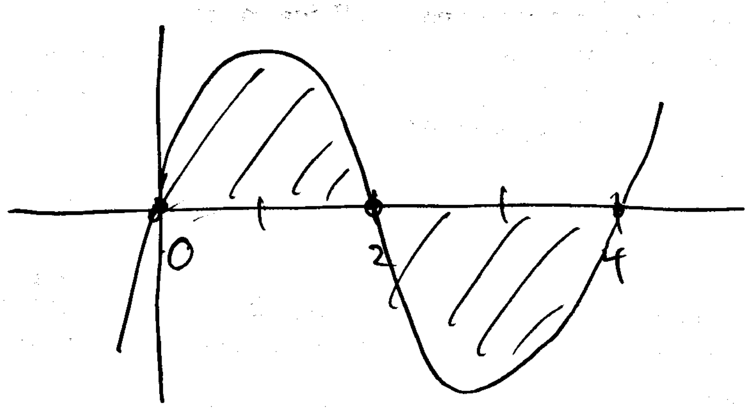
$= \int_0^4 x(x^2-6x+8)\,dx$

$= \int_0^4 (x^3-6x^2+8x)\,dx = \left(\frac{1}{4}x^4 - 2x^3 + 4x^2\right)\Big|_0^4$

$$=\left(\frac{1}{4}(4)^4-2(4)^3+4(4)^2\right)-(0)$$

$$=64-128+64=0$$



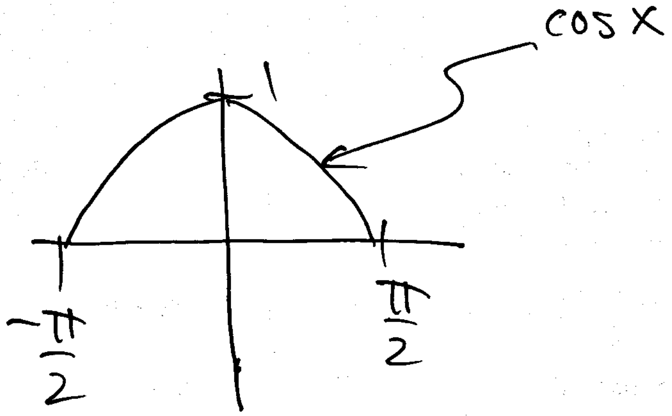


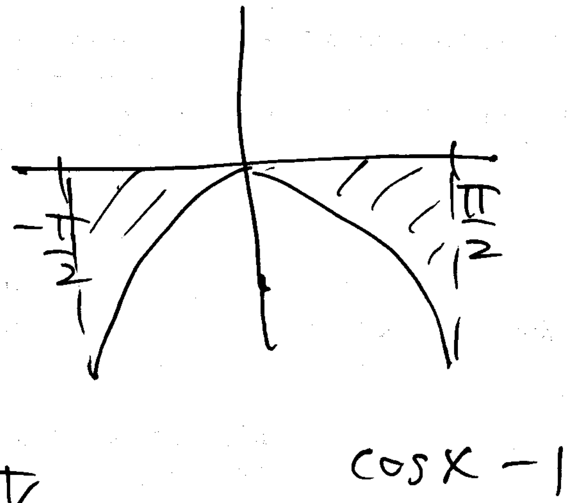
#38) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos x-1)\,dx=(\sin x-x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\left(\sin\frac{\pi}{2}-\frac{\pi}{2}\right)-\left(\sin\left(-\frac{\pi}{2}\right)-\left(-\frac{\pi}{2}\right)\right)$$

$$=\left(1-\frac{\pi}{2}\right)-\left(-1+\frac{\pi}{2}\right)$$

$$=1-\frac{\pi}{2}+1-\frac{\pi}{2}=2-\pi$$





$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x - 1)\,dx = 2\int_0^{\pi/2} (\cos x - 1)\,dx$$

$$= 2\,(\sin x - x)\Big|_0^{\pi/2} = 2\left(\sin\tfrac{\pi}{2} - \tfrac{\pi}{2} - 0\right)$$

$$= 2\left(1 - \tfrac{\pi}{2}\right) = 2 - \pi.$$