Quiz 11 4.8, 4.7 Tuesday

Exam 3 4.1–4.8 Wednesday

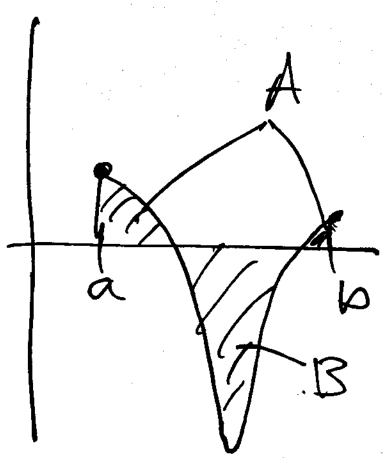
Review sessions still available.

$\int f(x)\,dx = F(x) + C$ — <u>most general anti-derivative</u> of $f(x)$ <u>or</u> the <u>indefinite integral</u> of $f(x)$.

where $F'(x) = f(x)$.

$\int_a^b f(x)\,dx \leftarrow$ <u>net</u> area under graph of $f(x)$ between $x=a$ and $x=b$.

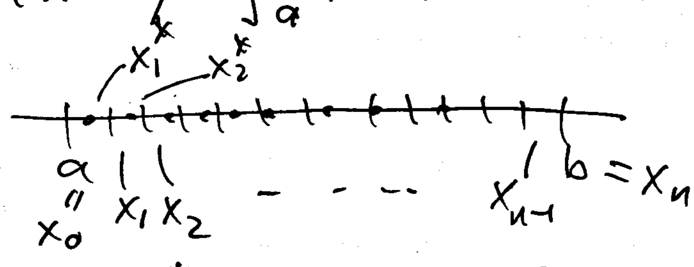
i.e., (area under graph above x-axis) − (area above graph below x-axis)



$\int_a^b f(x)\,dx = \text{area}(A) - \text{area}(B)$

Defining $\int_a^b f(x)\,dx$.

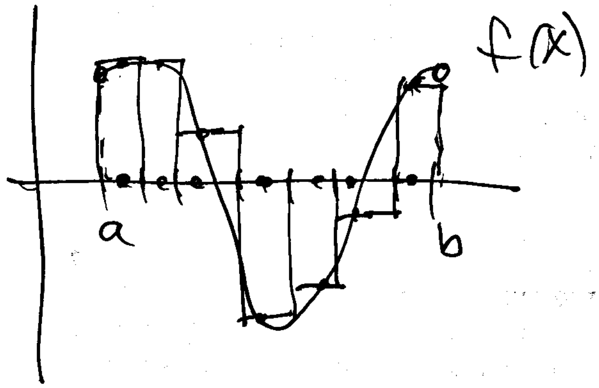
①



② Choose x_i^* in $[x_{i-1}, x_i]$ for $i = 1, 2, 3, \ldots, n$.

③ Form the sum

Riemann sum $\rightarrow \sum_{i=1}^{n} f(x_i^*)(x_i - x_{i-1}) = \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$



$f(x_i^*)\Delta x_i =$ "area" of rectangle above (or below) $[x_{i-1}, x_i]$.

(4) $$\int_a^b f(x)\,dx = \lim_{\underbrace{\Delta x \to 0}} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

↳ means the intervals in the partition get smaller and smaller.

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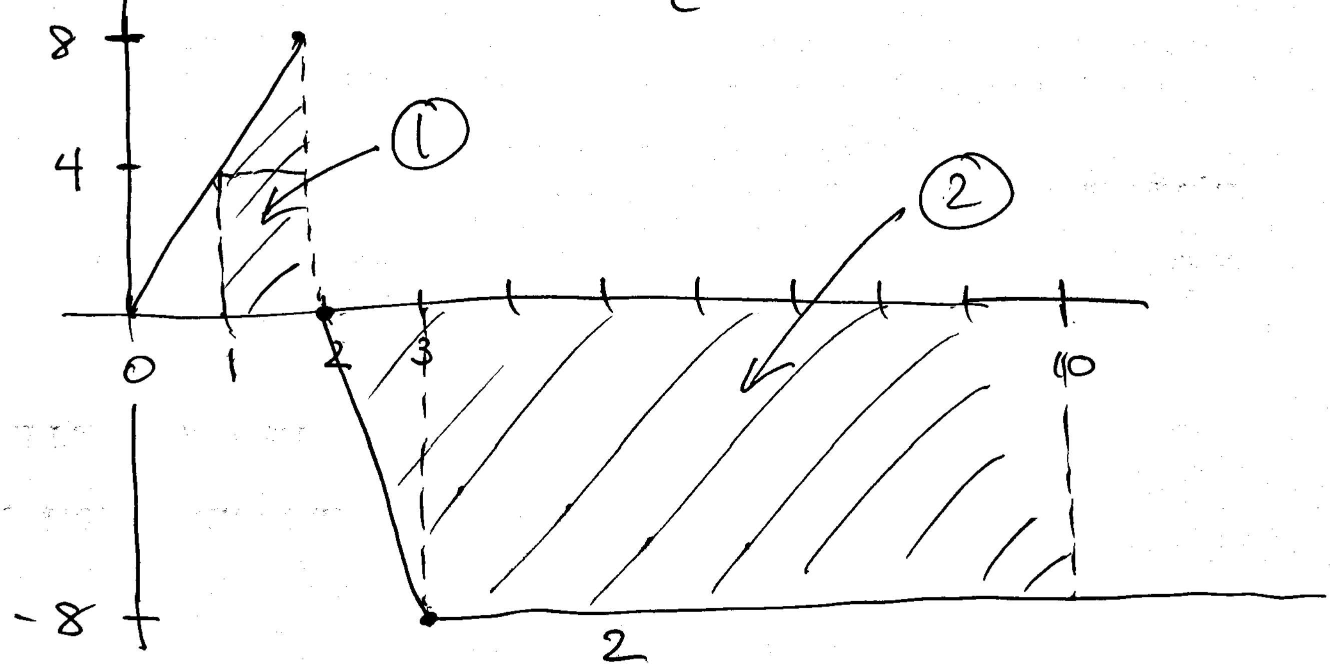
$$\int_1^{10} g(x)\,dx \qquad g(x) = \begin{cases} 4x & 0 \le x \le 2 \\ -8x+16 & 2 < x \le 3 \\ -8 & x > 3. \end{cases}$$



$$① = \text{area}(\square) + \text{area}(\Delta)$$

$$= 4 \cdot 1 + \tfrac{1}{2}(1 \cdot 4) = 6$$

$$② = \text{area}(\nabla) + \text{area}(\square)$$

$$= \tfrac{1}{2}(1 \cdot 8) + 7 \cdot 8$$

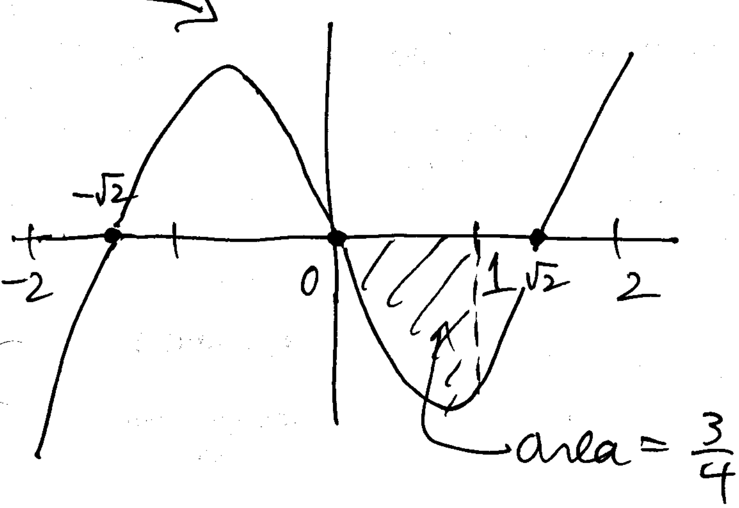
$$= 60$$

$$\int_1^{10} g(x)\,dx = 6 - 60 = -54$$

#43) $\int_0^1 (x^3 - 2x)\,dx = -\frac{3}{4}$

a. $\int_0^1 (4x - 2x^3)\,dx$ b. $\int_1^0 (2x - x^3)\,dx$

$$x^3 - 2x = x(x^2 - 2)$$



a. $-2(x^3-2x) = \cancel{2x^3-4x}\ 4x-2x^3$

$$\int_0^1 (4x-2x^3)\,dx = \int_0^1 (-2)(x^3-2x)\,dx$$

$$= (-2)\underbrace{\int_0^1 (x^3-2x)\,dx}_{} = -2\cdot\frac{-3}{4} = 3/2.$$

b. $$\int_1^0 (2x-x^3)\,dx = -\int_0^1 (2x-x^3)\,dx$$

$$= -\int_0^1 -(x^3-2x)\,dx = -(-1)\int_0^1 (x^3-2x)\,dx$$

$$= \int_0^1 (x^3-2x)\,dx = -\frac{3}{4}.$$

5.3 Fundamental Thm of Calculus.

Idea: Want to compute $\int_a^b f(x)\,dx$.

Principle:

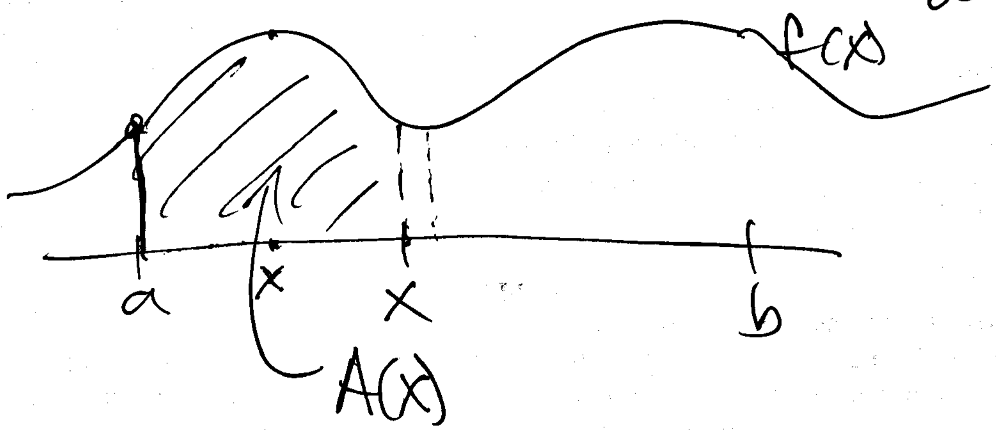
$$\begin{pmatrix}\text{area under the} \\ \text{curve } y=f(x) \\ \text{between } x=a \text{ + } x=b\end{pmatrix} = \begin{pmatrix}\text{Total change} \\ \text{in antiderivative} \\ \text{of } f(x) \text{ on } [a,b]\end{pmatrix}$$

we have seen this work for piecewise constant functions and linear functions.

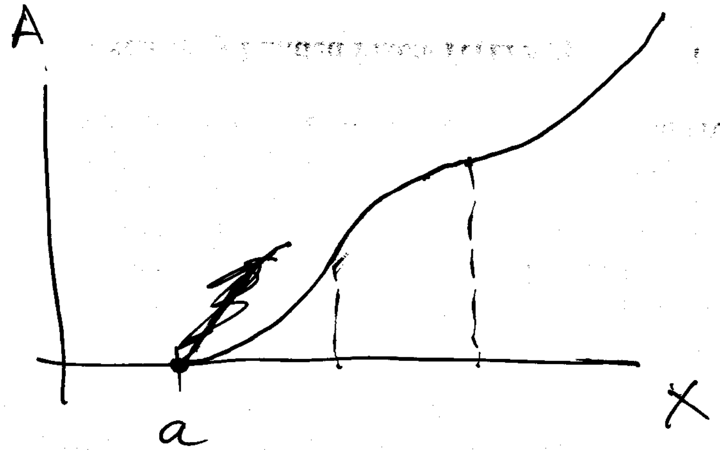
FTC is this principle for general functions

FTC Part I

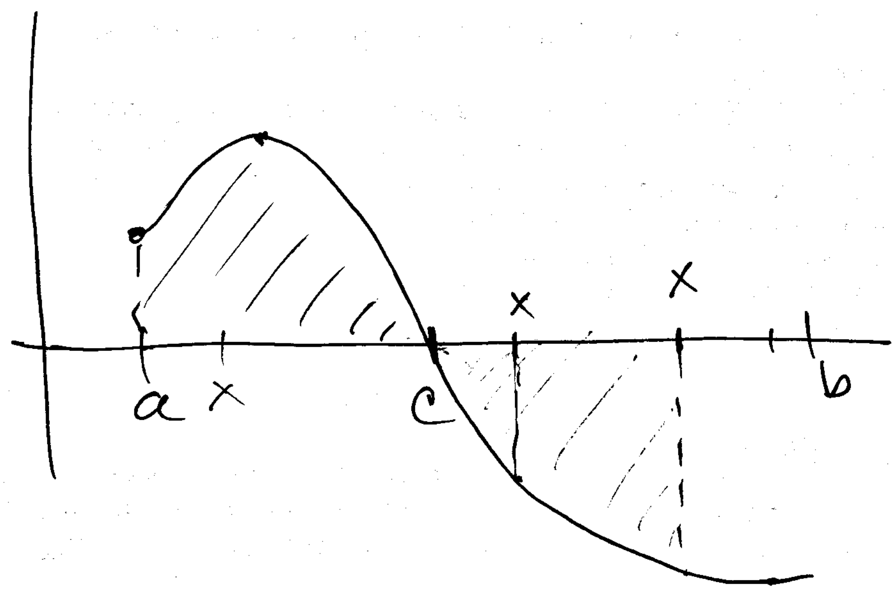
Given $f(x)$ on $[a,b]$, define the area function $A(x)$ by $A(x) = \int_a^x f(t)\,dt$



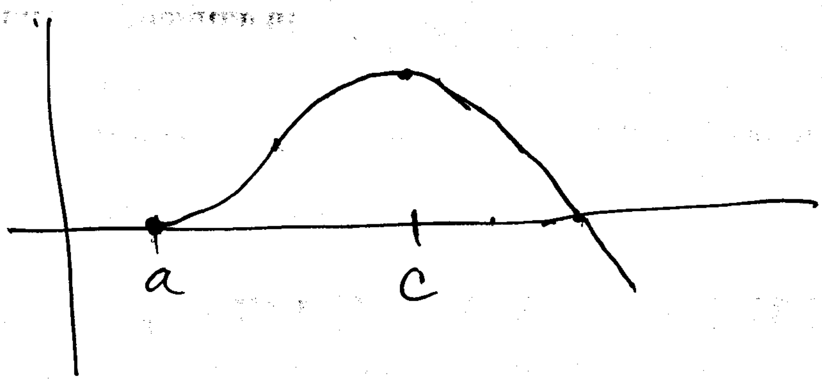
Then $A'(x) = f(x)$, i.e. $A(x)$ is an antiderivative of $f(x)$



$$A(a) = \int_a^a f(x)\,dx$$



f(x)



A(x)

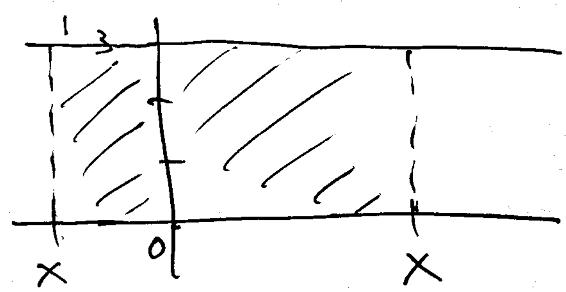
FTC Part II

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

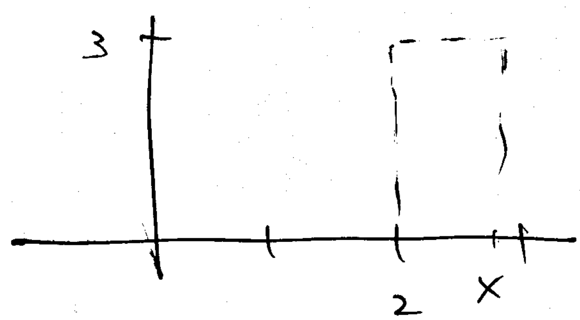
where $F'(x) = f(x)$, i.e. F is <u>any</u> antiderivative of $f(x)$.

Examples:

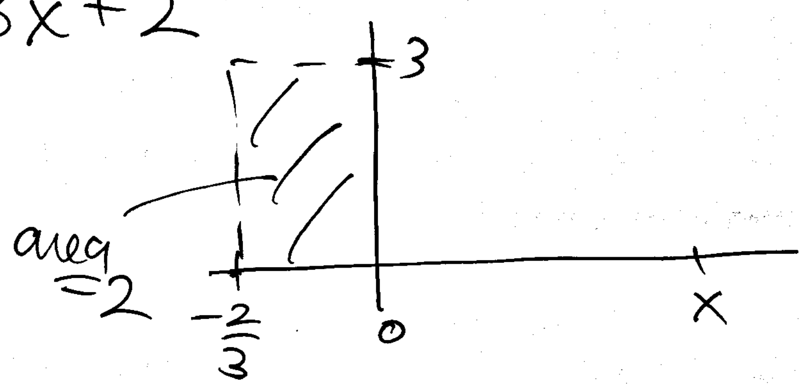
$$\int_0^x 3\,dt = 3x$$



$$\frac{d}{dx}(3x) = 3$$

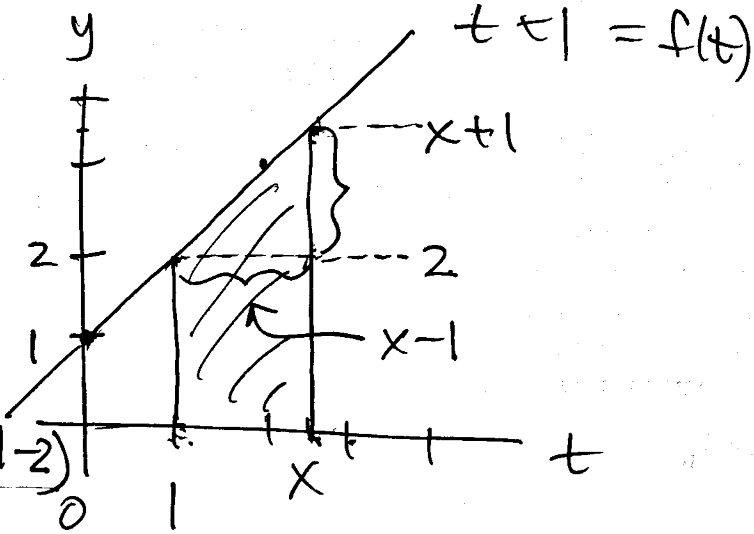


$$\int_{-\frac{2}{3}}^x 3\,dt = 3x + 2$$



$$\int_{\underline{\underline{a}}}^x 3\,dt = 3x + \underline{\underline{C}}$$

$$\boxed{\int_1^x \underbrace{(t+1)}_{f(t)}\,dt} = A(x)$$



$$= (x-1)(2) + \frac{1}{2}(x-1)(x+1-2)$$

$$x^2-2x+1$$

$$= 2x-2+\frac{1}{2}(x-1)^2$$

$$= 2x-2+\frac{1}{2}x^2 - x + \frac{1}{2}$$

$$= \frac{1}{2}x^2 + x - \frac{3}{2}$$

$$\frac{d}{dx}\underbrace{\left(\frac{1}{2}x^2+x-\frac{3}{2}\right)} = x+1$$

$$\equiv$$

$$\frac{dy}{dx}$$

$$\frac{d\left(\frac{1}{2}x^2+x-\frac{3}{2}\right)}{dx} = \frac{d}{dx}\left(\frac{1}{2}x^2+x-\frac{3}{2}\right)$$

eg $\int_{-2}^{1} (x^2 - x - 6)\,dx$

$$= \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x\right)\Big|_{x=-2}^{x=1}$$

$$= \left(\frac{1}{3}(1)^3 - \frac{1}{2}(1)^2 - 6(1)\right) - \left(\frac{1}{3}(-2)^3 - \frac{1}{2}(-2)^2 - 6(-2)\right)$$

$$= \frac{1}{3} - \frac{1}{2} - 6 - \left(-\frac{8}{3} - 2 + 12\right)$$

$$= \frac{1}{3} - \frac{1}{2} - 6 + \frac{8}{3} + 2 - 12$$

$$= 3 - 6 + 2 - 12 - \frac{1}{2} = -\frac{27}{2}$$