Quiz 10, 4.5, 4.6

Exam 3 - Wed 11-28 Chapter 4

Oral Reviews - Monday / Tuesday ~~11-26, 11-28~~
11-26 11-27

① Schedule will be posted online
② Email me with preferred times.

Antiderivatives: Given $f(x)$. Find $F(x)$ so that $F'(x) = f(x)$.

Notation: $\int f(x)\,dx = F(x) + c$

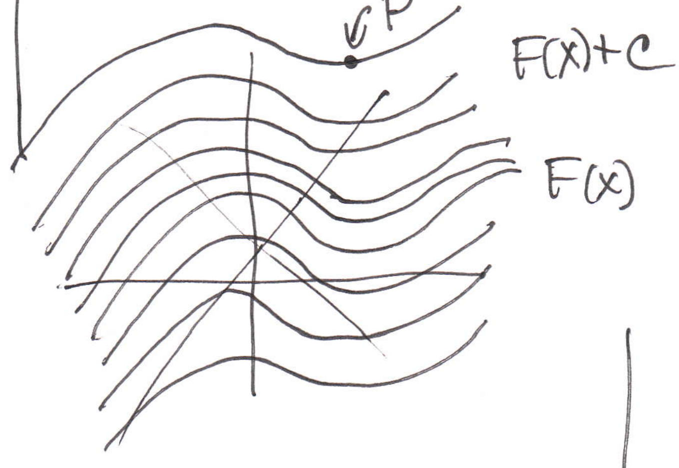
e.g.: $\int (2x^{-3}+2)\,dx = \int 2x^{-3}\,dx + \int 2\,dx$

$= 2\int x^{-3}\,dx + \int 2\,dx$

$= 2\left(-\frac{1}{2}x^{-2}\right) + 2x + c$

$= -x^{-2} + 2x + c$

$\int f(x)\,dx = F(x) + c$



eg $\int \left(\frac{1}{3x} + \cos(5x) \right) dx$

$$= \int \frac{1}{3x}\,dx + \int \cos(5x)\,dx$$

$$= \frac{1}{3}\int \frac{1}{x}\,dx + \frac{1}{5}\sin(5x)$$

$$= \frac{1}{3}\ln|x| + \frac{1}{5}\sin(5x) + C$$

Why not! $\int \frac{1}{3x}\,dx \neq \ln|3x|$

$$\frac{d}{dx}\ln|3x| = \frac{1}{\cancel{3}x}\cdot\cancel{3} = \frac{1}{x}$$

$$\boxed{\frac{1}{3x} = (3x)^{-1} = 3^{-1}x^{-1} = \frac{1}{3}x^{-1}}$$

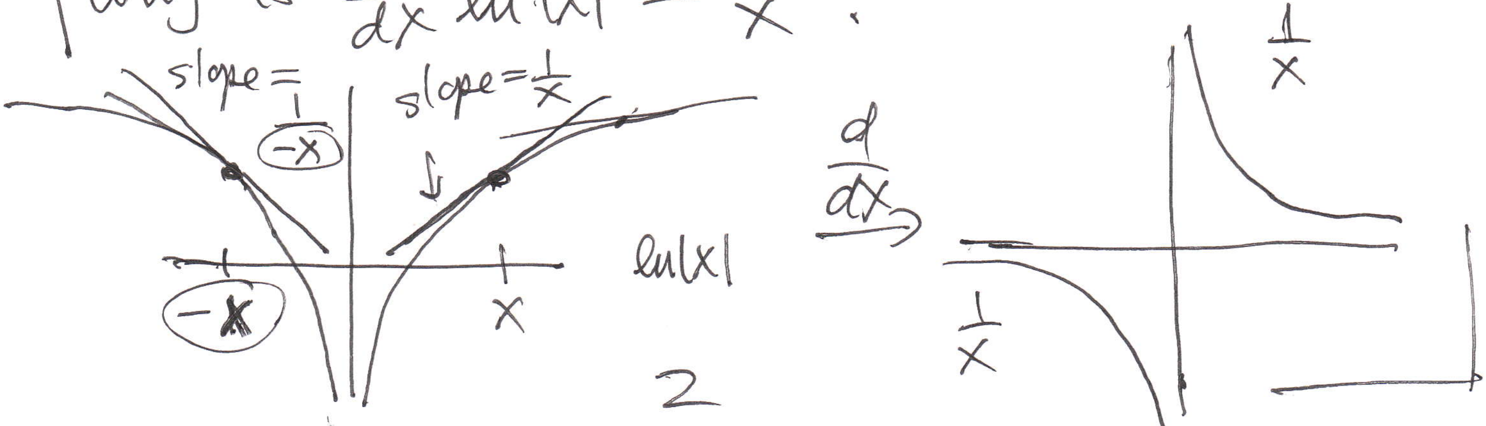
$$\ln|3x| = \ln(3) + \ln|x|$$

$$\frac{d}{dx}\ln|3x| = \frac{d}{dx}(\ln 3 + \ln|x|)$$

$$= 0 + \frac{1}{x} = \frac{1}{x}$$

Why $\ln|x|$ not $\ln(x)$?

Why is $\frac{d}{dx}\ln|x| = \frac{1}{x}$?



eg $\int \left(\sec^2(3x) + e^{x/2}\right) dx$

$= \frac{1}{3}\tan(3x) + 2e^{x/2} + C$

$\boxed{e^{x/2} \xrightarrow{\frac{d}{dx}} \frac{1}{2}e^{x/2}}$

~~$\int \ln(x)\, dx$~~ $\frac{d}{dx}\log_{10}(x) = \frac{1}{x\ln(10)}$

$\int \frac{1}{x\ln(10)}\, dx = \frac{1}{\ln(10)}\int \frac{1}{x}\, dx = \frac{1}{\ln(10)}\ln|x| + c$

$= \frac{\ln|x|}{\ln(10)} + c$

Initial Value Problem.

$\frac{dy}{dx} = f(x)$ $\quad y(x_0) = y_0$. Find $y(x)$.

satisfying both conditions

e.g. $\dfrac{dy}{dx} = 9x^2 - 4x$, $y(-1) = 0$

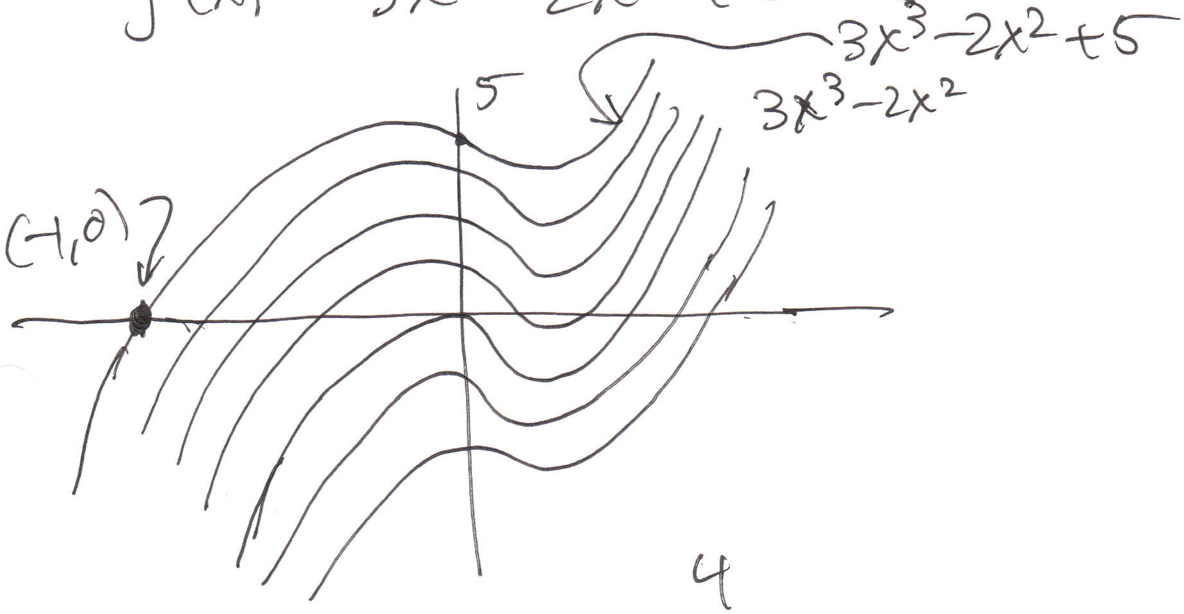
Find C

$$y(x) = \int (9x^2 - 4x)\,dx$$
$$= 9 \cdot \tfrac{1}{3}x^3 - 4 \cdot \tfrac{1}{2}x^2 + C$$
$$= 3x^3 - 2x^2 + C$$

$$0 = y(-1) = 3(-1)^3 - 2(-1)^2 + C$$
$$= -3 - 2 + C$$
$$= -5 + C$$

$$0 = -5 + C \longrightarrow \underline{\underline{C = 5}}$$

$$y(x) = 3x^3 - 2x^2 + 5$$



eg #45 $g'(x) = 7x\left(x^6 - \frac{1}{7}\right)$; $g(1) = 2$

$$g(x) = \int 7x\left(x^6 - \frac{1}{7}\right)dx = \int (7x^7 - x)\,dx$$

$$= \frac{7}{8}x^8 - \frac{1}{2}x^2 + C$$

$$2 = g(1) = \frac{7}{8} - \frac{1}{2} + C = \frac{3}{8} + C$$

$$C = 2 - \frac{3}{8} = \frac{13}{8}$$

$$g(x) = \frac{7}{8}x^8 - \frac{1}{2}x^2 + \frac{13}{8}.$$

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eg. #81) Flow rate $Q'(t) = (.1)(100 - t^2)\ \frac{\text{gal}}{\text{min}}$

(a) $Q(t) = \int (.1)(100 - t^2)\,dt$

$$= (.1)\int (100 - t^2)\,dt = .1\left(100t - \frac{1}{3}t^3\right) + C$$

$$= 10t - \frac{1}{30}t^3 + C$$

$Q(0) = 0.$ $\quad 0 = Q(0) = 10(0) - \frac{1}{30}(0)^3 + C$

$Q(t) = 10t - \frac{1}{30}t^3$ $\quad = C$

5

c. Amt of water out of tank in 10 min:

$$Q(10) = 10(10) - \frac{1}{30}(10)^3$$

$$= 100 - \frac{100}{3} = \frac{2}{3}(100) \approx 67 \text{ gal.}$$

How much flows out between 5 and 10 minutes?

$$Q(10) - Q(5) = \frac{2}{3}(100) - \left(50 - \frac{\overset{25}{\cancel{125}}}{\underset{6}{\cancel{30}}}\right)$$

$$= \frac{400}{6} - \frac{300}{6} + \frac{25}{6} = \frac{125}{6} \approx 20.8 \text{ gal}$$

5.1 Approximating areas under curves.

<u>Idea:</u> $f'(x)$ – slope of tangent line to graph of f

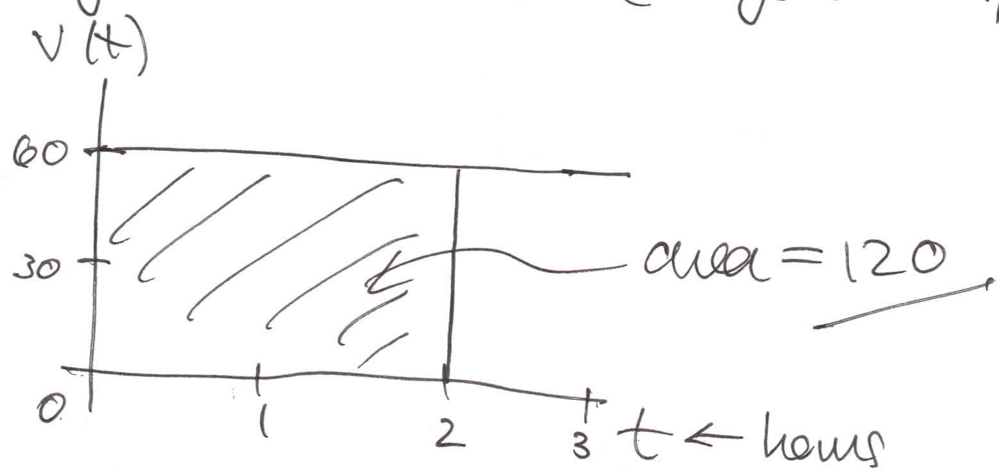
Is there a way to visualize antiderivative?

Yes: Is related to area.

<u>Example:</u> Suppose we have $v(t)$ = velocity at t

$\int v(t)\,dt = s(t)$ position function.

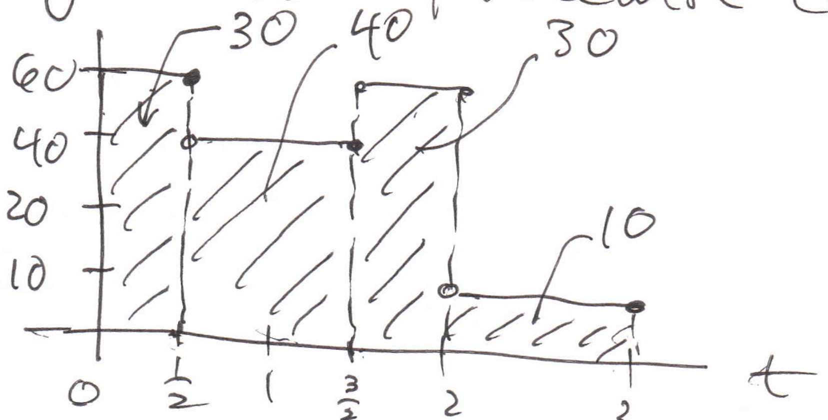
Say $v(t)$ = const (say 60 mph)



How far have I travelled between $t=0$, $t=2$?

~~s(2.5)~~ $s(2) - s(0) = 120$

Say $v(t)$ – piecewise constant



How far have I traveled betw $t=0$, $t=3$?

$30+40+30+10$

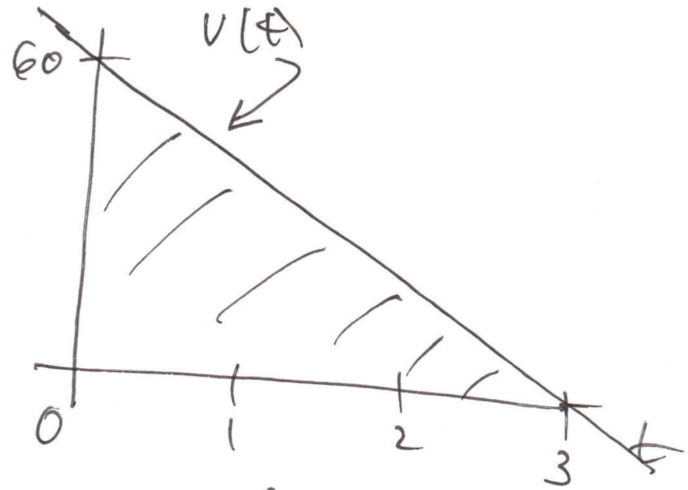
$= 110$ mi.

= area under

velocity curve

Also say: if $s(t)$ is the position function then $s(3)-s(0)=110$.

Say $v(t)$ is linear.



Area under $v(t) =$
$\frac{1}{2}(3)(60) = 90$ mi.

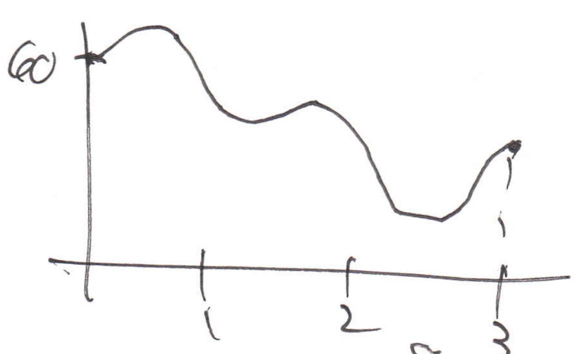
$v(t) = -20t+60$

$$s(t) = \int(-20t+60)\,dt = -10t^2+60t+c$$

How far betw $t=0$, $t=3$?

$$s(3)-s(0) = -10(3)^2+60(3)+\cancel{c}$$
$$-\left(\cancel{-10(0)^2}+\cancel{60(0)}+\cancel{c}\right)$$
$$= -90+180 = 90 \,//.$$

Q: What about arbitrary $v(t)$?



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