Quiz 9 – 4.3, 4.4
Exam 3 – Wed. 11-28 (Chapter 4)

Indeterminate forms:

$$\underbrace{\frac{0}{0}, \frac{\infty}{\infty}}, 0\cdot\infty, 1^{\infty}, \infty^0, 0^0, \infty-\infty$$

↳ can be evaluated by L'Hopital's Rule.

e.g. $\lim\limits_{x\to\infty} x^2e^{-x}$ form $\infty\cdot 0$

2 choices: $\frac{0}{0} \longrightarrow \lim\limits_{x\to\infty} \frac{e^{-x}}{\left(\frac{1}{x^2}\right)} = \lim\limits_{x\to\infty} \frac{e^{-x}}{x^{-2}}$ ①

$\frac{\infty}{\infty} \longrightarrow \lim\limits_{x\to\infty} \frac{x^2}{\left(\frac{1}{e^{-x}}\right)} = \lim\limits_{x\to\infty}\frac{x^2}{e^x}$ ②

①: $\lim\limits_{x\to\infty} \frac{e^{-x}}{x^{-2}} = \lim\limits_{x\to\infty} \frac{+e^{-x}}{+2x^{-3}}$ ← form: $\frac{0}{0}$

$= \lim\limits_{x\to\infty} \frac{+e^{-x}}{+6x^{-4}}$ ← form $\frac{0}{0}$ hopeless

(2): $\lim\limits_{x\to\infty} \dfrac{x^2}{e^x} = \lim\limits_{x\to\infty} \dfrac{2x}{e^x}$ ← form: $\dfrac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \dfrac{2}{e^x} = 0$

e.g. $\lim\limits_{x\to\infty} x^{10} e^{-x} = \lim\limits_{x\to\infty} \dfrac{x^{10}}{e^x} = \lim\limits_{x\to\infty} \dfrac{10x^9}{e^x}$

$= \lim\limits_{x\to\infty} \dfrac{10\cdot 9x^8}{e^x} = \cdots = \lim\limits_{x\to\infty} \dfrac{10\cdot 9\cdot 8\cdot 7\cdot \cdots \cdot 3\cdot 2\cdot 1}{e^x}$

$= 0.$

In fact: $\lim\limits_{x\to\infty} x^n e^{-x} = 0$ for any $n>0$.

"Exponential decay beats polynomial growth."

e.g. $\lim\limits_{x\to 0^+} x \ln(x) \quad (0\cdot\infty)$

$= 0$

$\lim\limits_{x\to 0^+} x^{1/10} \ln(x) = \lim\limits_{x\to 0^+} \dfrac{\ln(x)}{x^{-1/10}} = \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{10} x^{-\frac{11}{10}}}$ $\quad \dfrac{\infty}{\infty}$

$= \lim\limits_{x\to 0^+} (-10) \dfrac{x^{11/10}}{x} = \lim\limits_{x\to 0^+} (-10) \dfrac{}{x^{1/10}} =$

$= \lim\limits_{x\to 0^+} (-10) x^{1/10} = 0.$

In fact $\lim_{x\to 0^+} x^\alpha \ln(x) = 0$ for all $\alpha > 0$.

e.g. $\lim_{x\to\infty} \frac{\ln(x)}{x}$ form: $\frac{\infty}{\infty}$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

In fact $\lim_{x\to\infty} \frac{\ln x}{x^\alpha} = 0$ for any $\alpha > 0$.

$\lim_{x\to\infty} \left(x - \sqrt{x^2+1}\right)$ form: $\infty - \infty$.

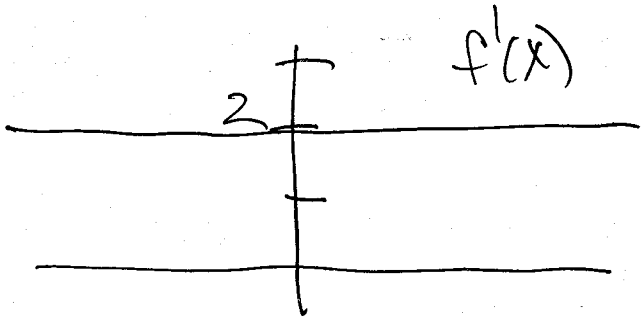
$$= \lim_{x\to\infty} \left(x-\sqrt{x^2+1}\right) \frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x^2-(x^2+1)}{x+\sqrt{x^2+1}}$$

$$= \lim_{x\to\infty} \frac{-1}{x+\sqrt{x^2+1}} = 0$$

4.8 Antiderivatives

Problem: Given $f'(x)$ find $f(x)$.

e.g. $f'(x) = 2 \longrightarrow f(x) = 2x$

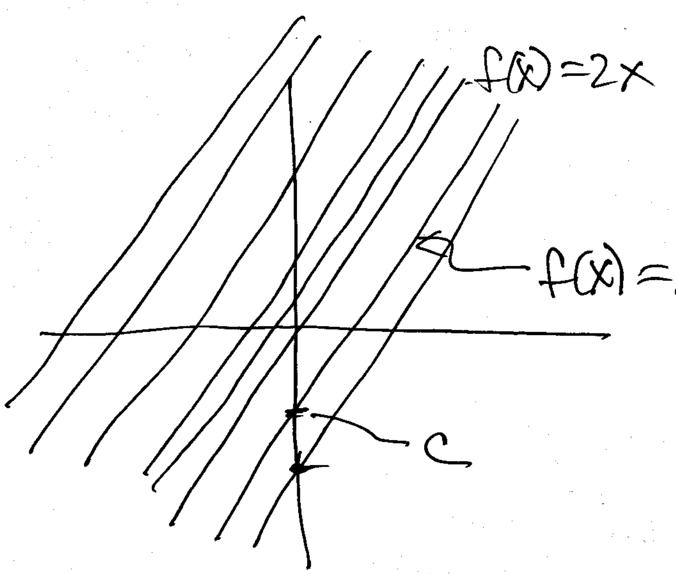


only solution?

$f(x) = 2x + 3$

$f(x) = 2x - 5$

etc...

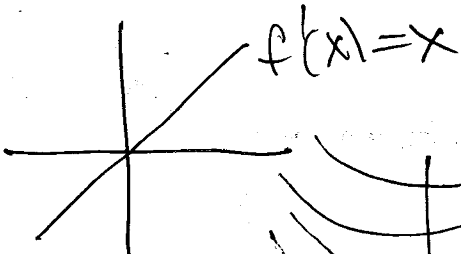


General solution

$f(x) = 2x + C$

any real c.

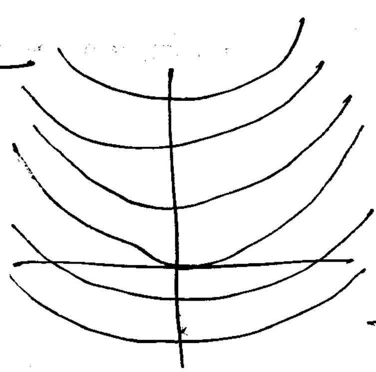
e.g. $f'(x) = x$



$f(x) = \frac{1}{2}x^2$ works

$f(x) = \frac{1}{2}x^2 + \frac{5}{3}$ works

$f(x) = \frac{1}{2}x^2 + C$ works.



<u>Def</u>: A function $F(x)$ is an <u>antiderivative</u> of $f(x)$ if $F'(x) = f(x)$.

<u>Thm</u>: If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ then $F(x) = G(x) + c$ for some constant c.

Antiderivative rules:

1. $f(x) = x^r$ $\quad F(x) = \frac{1}{r+1} x^{r+1} + c$
2. $f(x) = \sin(x)$ $\quad F(x) = -\cos(x) + c$
3. $f(x) = \cos(x)$ $\quad F(x) = \sin(x) + c$
4. $f(x) = \tan(x)$ $\quad F(x) =$ not easy
5. $f(x) = \sec^2(x)$ $\quad F(x) = \tan(x) + c$
6. $f(x) = \sec(x)$ $\quad F(x) =$ wait until 114
7. $f(x) = e^x$ $\quad F(x) = e^x + c$
8. $f(x) = 2^x$ $\quad F(x) = \frac{1}{\ln 2} 2^x + c$

$\vdots$

Integral notation:

The most general antiderivative of $f(x)$ is denoted

$$\int f(x)\,dx = F(x) + C$$

where $F'(x) = f(x)$.

(a) $\int \frac{1}{x}\,dx = \ln|x| + C$