Quiz 8 - 3.8, 3.9

Exam 2 - 10/31 3.1-3.10,

Oral Reviews on 10/29, 10/30

Using f', f'' to describe functions and graphs.

- f' determines whether f is increasing or decreasing
- f'' determines whether f is concave up or down
- local extrema always occur at critical points ($f' = 0$ or f' undefined.)

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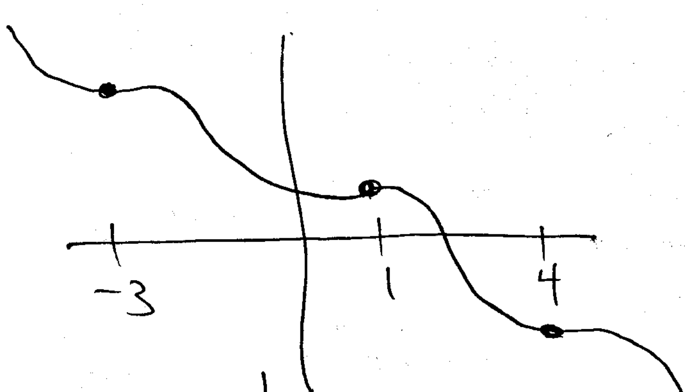
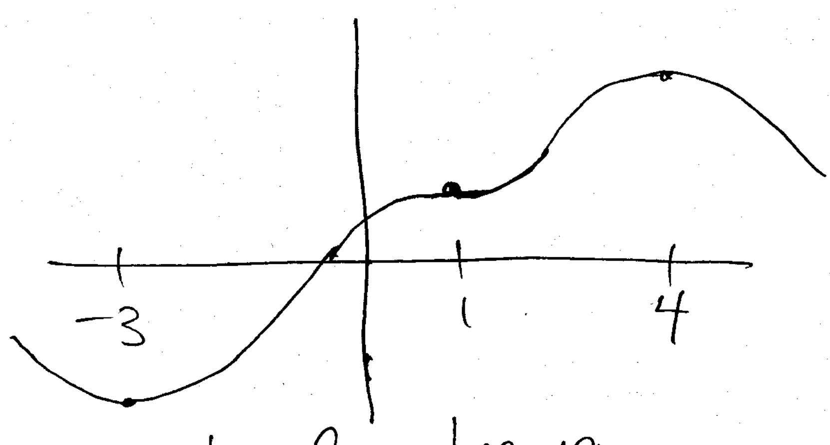
e.g. #37 p255

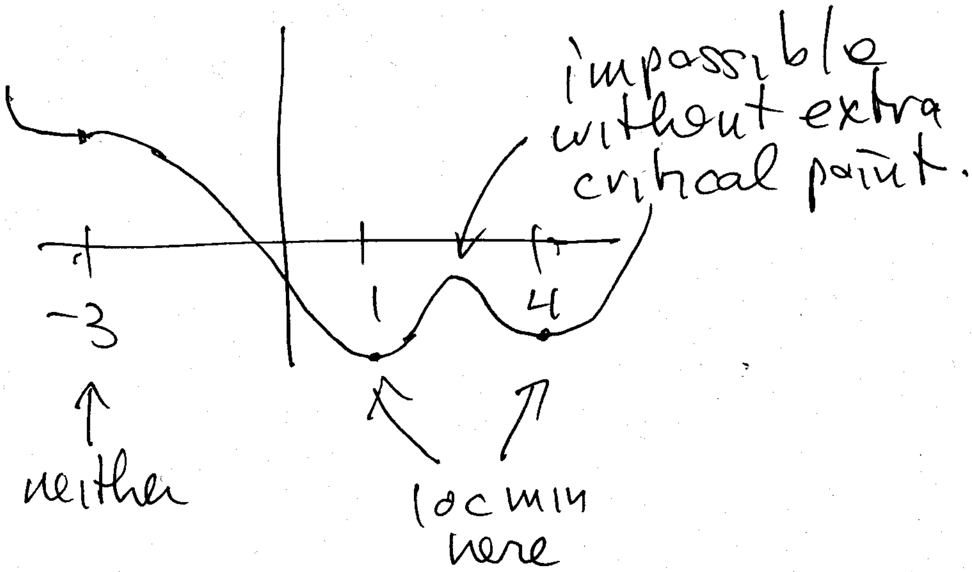
(a) True. If f has any local extrema, they would be at $x = -3, 1,$ or 4, but f does not have to have local extrema there.



no extrema.



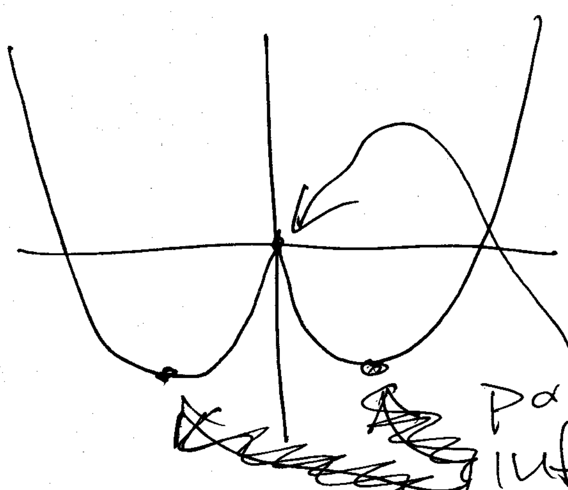
2 local extrema.



(b) True. Any inflection points must be at $x=-2$ or 4 but these points do not have to be inflection points.

This means a point where concavity changes from up to down or down to up.

e.g. $f(x)=x^{2/3}(x^2-4)$



possible inflection pts and local maximum ~~minimums~~

but f is concave up everywhere.

$$f'(x) = x^{2/3}(2x) + \frac{2}{3}x^{-1/3}(x^2-4)$$

$$= 2x^{5/3} + \frac{2}{3}x^{5/3} - \frac{8}{3}x^{-1/3}$$

$$= \frac{6x^2+2x^2-8}{3x^{1/3}} = \frac{8x^2-8}{3x^{1/3}}$$

$$= \frac{8}{3}\left(\frac{x^2-1}{x^{1/3}}\right)$$

$$f''(x) = \frac{8}{3}\,\frac{x^{1/3}\cdot 2x - (x^2-1)\frac{1}{3}x^{-2/3}}{x^{2/3}}\;\frac{x^{2/3}}{x^{2/3}}$$

$$= \frac{8}{9}\,\frac{6x^2-x^2+1}{x^{4/3}}$$

$$= \frac{8}{9}\,\frac{5x^2+1}{x^{4/3}}.$$ Always ≥ 0

except when $x=0$

There f'' is undefined.

4.4 Optimization Problems.

#7) x, y are our numbers.

$x + y = 23$ (constraint)

Maximize $P = xy$

$x = 10 \quad y = 13 \quad P = 130$

$x = 9 \quad y = 14 \quad P = 126$

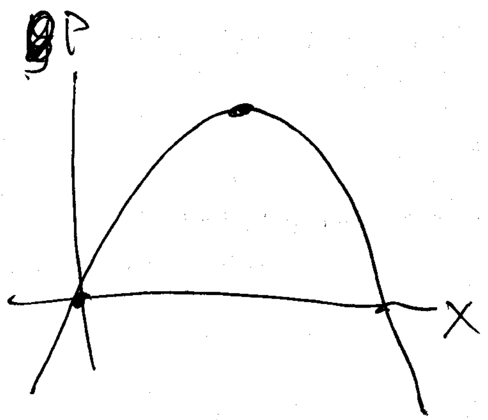
$x = 11 \quad y = 12 \quad P = 132$ ← best guess yet!

$x = 8 \quad y = 15 \quad P = 120$

$x = 11.25 \quad y = 11.75 \quad P = 132.1875$

~~Want~~ Need to write P as a function of one variable. $y = 23 - x$

$$P = x(23 - x) = 23x - x^2$$



$$P' = 23 - 2x$$

$$23 - 2x = 0$$

$$x = \frac{23}{2} = 11.5$$

$$y = 23 - 11.5 = 11.5$$

$$P = xy = (11.5)(11.5) = 132.25$$

#9) x, y numbers.

$$xy = 50$$

Minimize $S = x + y$.

$x = 10 \quad y = 5 \quad S = 15$

$x = 15 \quad y = \frac{50}{15} = \frac{10}{3} \quad S = 15 + \frac{10}{3} = \frac{55}{3} \approx 18.33$

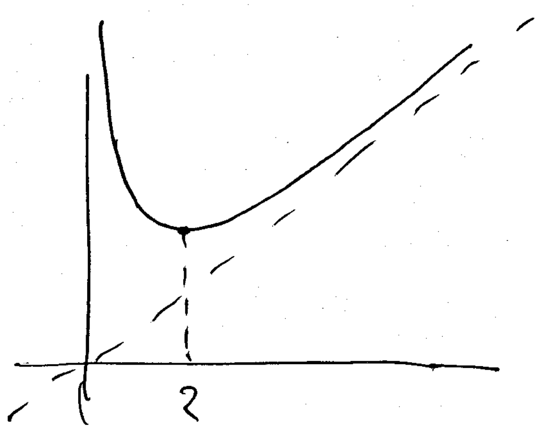
$x = 12 \quad y = \frac{50}{12} = \frac{25}{6} \quad S = 12 + \frac{25}{6} = \frac{72+25}{6} = \frac{97}{6}$

≈ 16.167

$x = 9 \quad y = \frac{50}{9} \quad S = \frac{81}{9} + \frac{50}{9} = \frac{131}{9} \approx 14.56$

$x = 8 \quad y = \frac{50}{8} = \frac{25}{4} \quad S = \frac{57}{4} = 14.25$

$$y = \frac{50}{x} \longrightarrow S = x + \frac{50}{x}$$



$$S' = 1 + \left(-\frac{50}{x^2}\right)$$

$$= 1 - \frac{50}{x^2}$$

$$1 - \frac{50}{x^2} = 0 \rightarrow x^2 = 50$$

$x = \sqrt{50}$

≈ 7.07

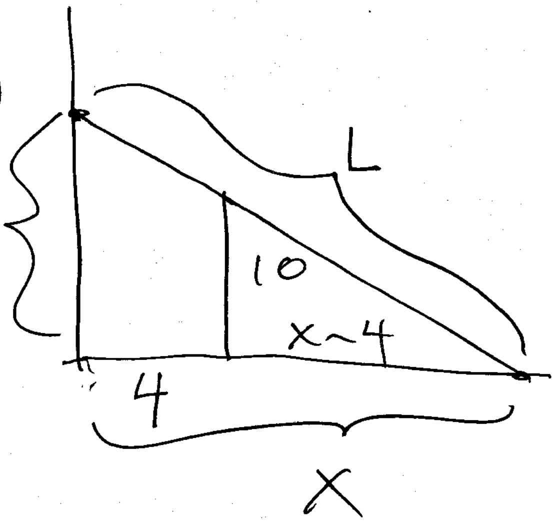
$x = \sqrt{50}$

$y = \sqrt{50}$

$S = \sqrt{50} + \sqrt{50} = 2\sqrt{50} \approx 14.14$

5

#16)



Minimize L

$L = (x^2+y^2)^{1/2}$

Easier to minimize

$L^2 = x^2+y^2$. Call it

$S = x^2+y^2 \leftarrow$ minimize this.

Find constraint.

$$\frac{y}{10} = \frac{x}{x-4} \longrightarrow y = \frac{10x}{x-4}$$

$$S = x^2 + \left(\frac{10x}{x-4}\right)^2 = x^2 + \frac{100x^2}{(x-4)^2}$$

Ans. $x = (400)^{1/3} + 4$

$$S' = 2x + \frac{200x(x-4)^{\cancel{2}} - 2\cancel{(x-4)}\cdot 100x^2}{(x-4)^{\cancel{4}3}}$$

$$= 2x + \frac{200x(x-4) - 200x^2}{(x-4)^3}$$

$$= 2x + \frac{-800x}{(x-4)^3}$$

$$2\cancel{x} = \frac{800\cancel{x}}{(x-4)^3} \longrightarrow 2(x-4)^3 = 800$$

$(x-4)^3 = 400$

$x = 400^{1/3} + 4 \approx 11.37$