Quiz 7 3.6, 3.7

Exam 2 – Wed 10/31 3.1–3.10

What do derivatives tell us about graphs of functions?

- find local extrema (at critical point where $f'=0$ or f' is not defined)
- whether graph is increasing ($f'>0$) or decreasing ($f'<0$).

$f'>0$ $f'<0$ — c is a local max, $f'(c)=0$

$f'<0$ $f'>0$ — c is a local min, $f'(c)=0$

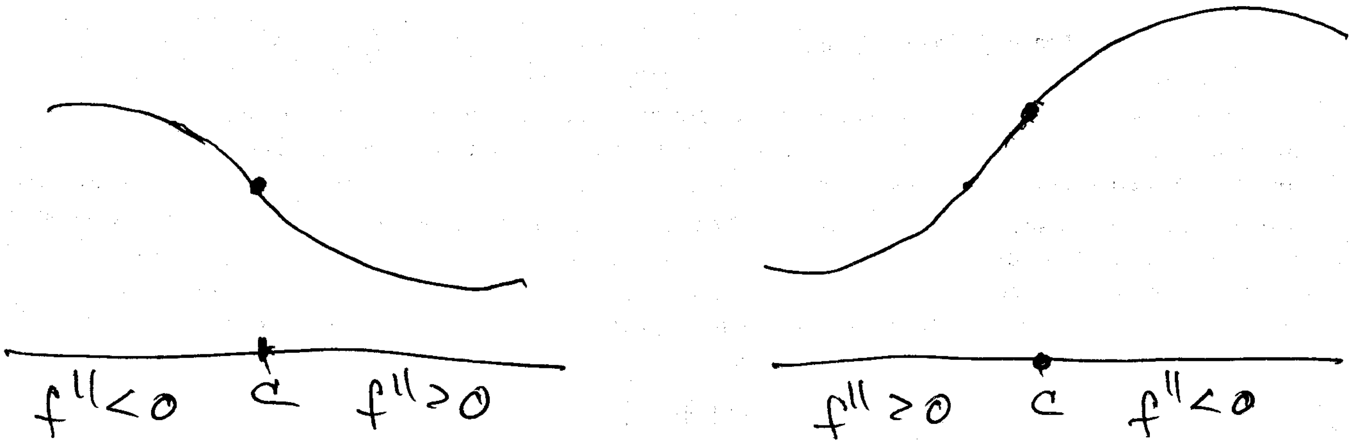
- f'' tells us concavity of graph.

$f''>0$ concave up ($\cup$), $f''<0$ concave down ($\frown$)

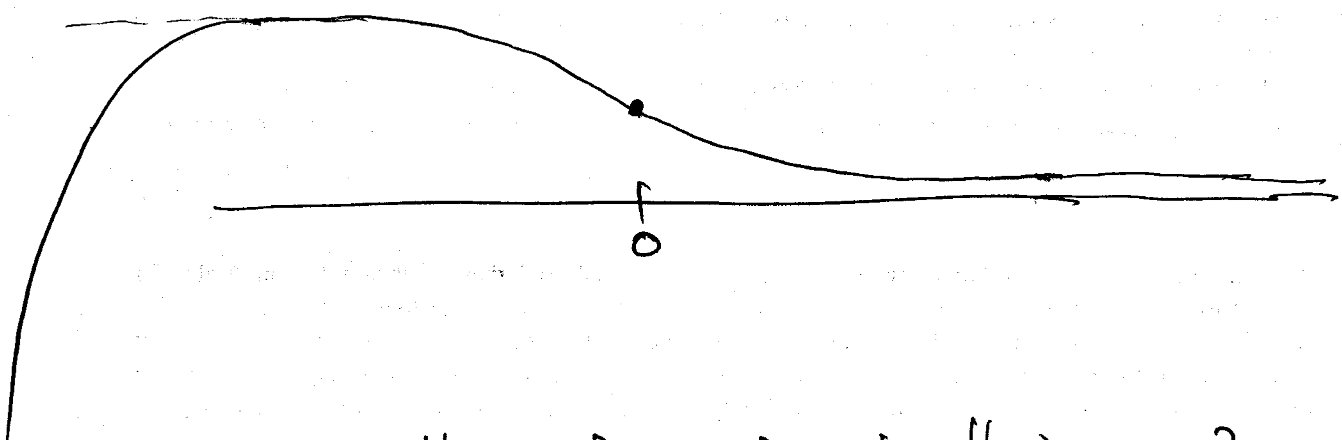
<u>Second Derivative Test:</u> Suppose $f'(c)=0$ (i.e. c is a critical point) and suppose

1. $f''(c)>0$ then c is a local minimum
2. $f''(c)<0$ then c is a local maximum
3. $f''(c)=0$ then you dont know.

<u>Def</u>: Suppose $f''(x)$ changes sign at $x=c$ (i.e. from $f''>0$ to $f''<0$ or $f''<0$ to $f''>0$). Then we call c an <u>inflection point</u> of f



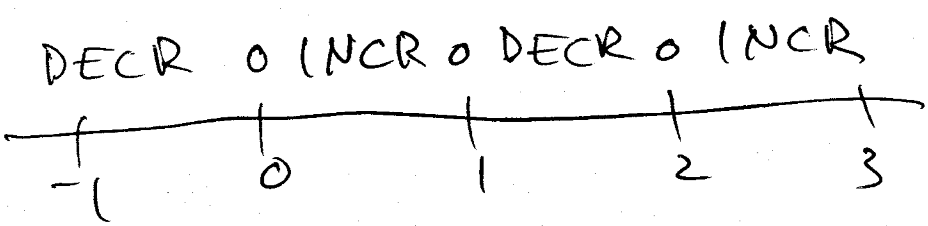
#45) p243 $f'(x)<0$ + $f''(x)<0$ on $(-\infty, 0)$
$f'(x)<0$ and $f''(x)>0$ on $(0,\infty)$



<u>e.g.</u> $g(x) = x^4 - 4x^3 + 4x^2$

<u>Rel:</u> $g'(x) = 4x^3 - 12x^2 + 8x$

crit pts $x=0$, $x=1$, $x=2$



$g''(x) = 12x^2 - 24x + 8$

$12x^2 - 24x + 8 = 0$

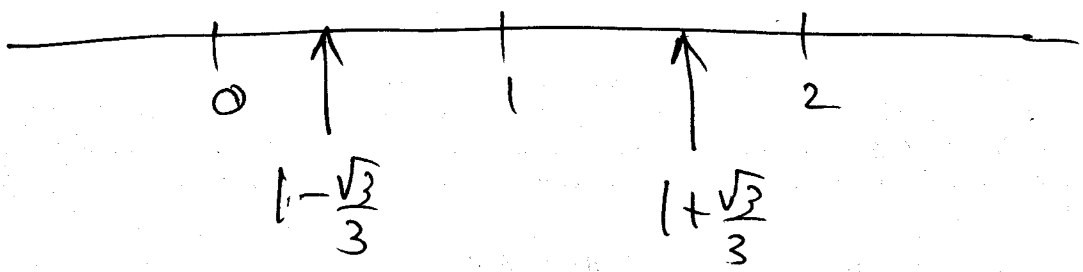
$4(3x^2 - 6x + 2) = 0$

$\cancel{4(3x\quad)(x\quad)=0}$

$$x = \frac{6 \pm \sqrt{36-24}}{6}$$

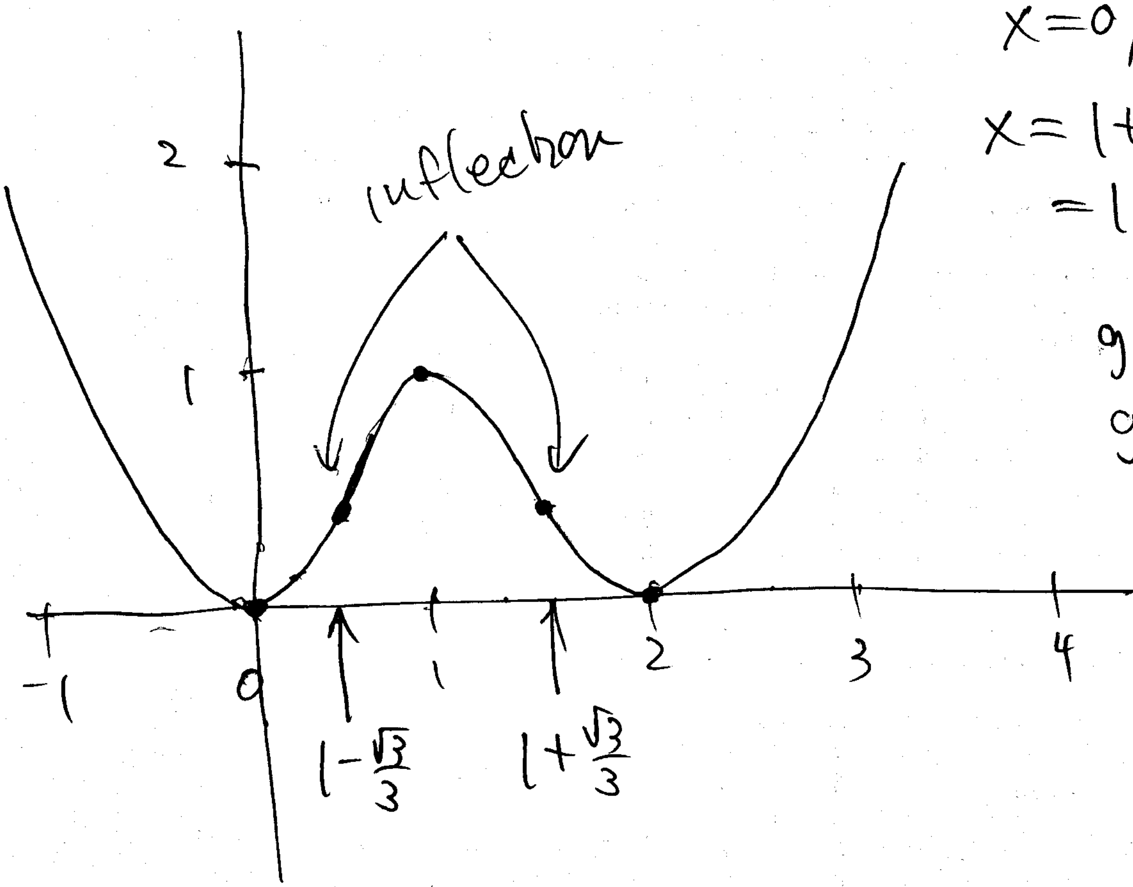
$$x = \frac{6 \pm 2\sqrt{3}}{6} = 1 \pm \frac{\sqrt{3}}{3} \begin{cases} \nearrow x \approx 1.58 \\ \searrow x \approx .42 \end{cases}$$

CONC UP 0 CONC DN 0 CONC UP. $\quad g''$



$g''(0) = 8 > 0 \quad g''(1) = 12 - 24 + 8 < 0 \quad g''(2) = 4(12 - 12 + 2) > 0$

Sketch graph of $g(x)$.



$x = 0, 1, 2$ critpts.

$x = 1 + \frac{\sqrt{3}}{3} \approx 1.58$ inflection

$= 1 - \frac{\sqrt{3}}{3} \approx .42$ points

$g(0) = 0$

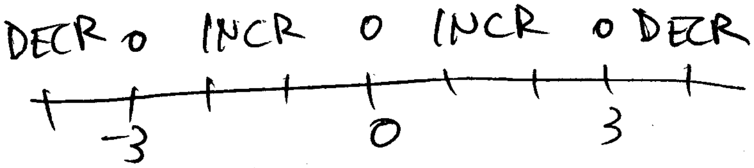
$g(1) = 1$

$g(2) = 16 - 32 + 16$
$= 0$

e.g. $f(t) = 15t^3 - t^5$

$f'(t) = 45t^2 - 5t^4$

critical pts: $t = 0, -3, 3$



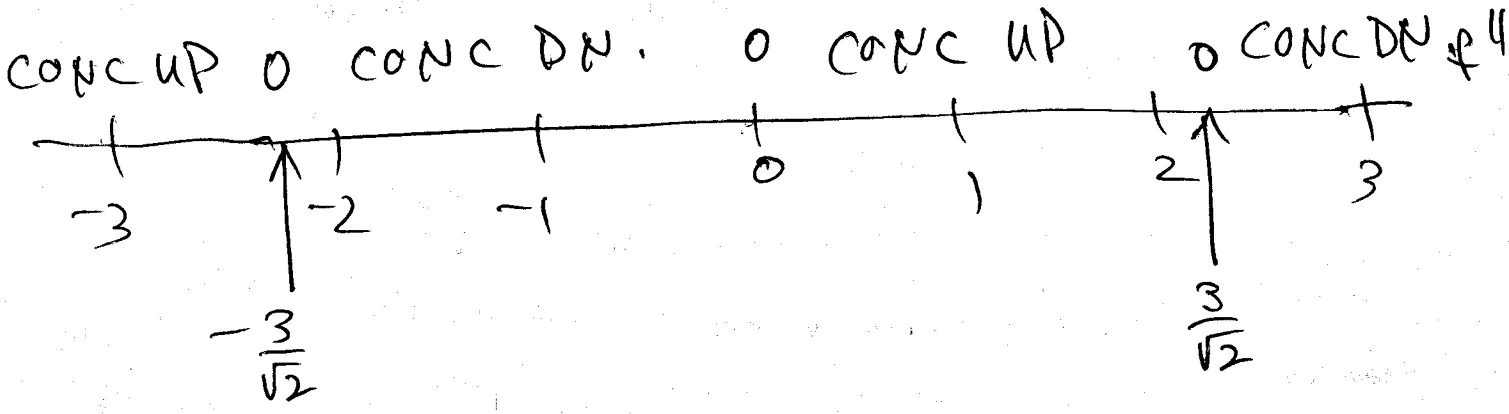
$f''(t) = 90t - 20t^3$

$90t - 20t^3 = 0$

$10t(9 - 2t^2) = 0$

$t = 0, \; t = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}, \; t = \frac{-3}{\sqrt{2}}$

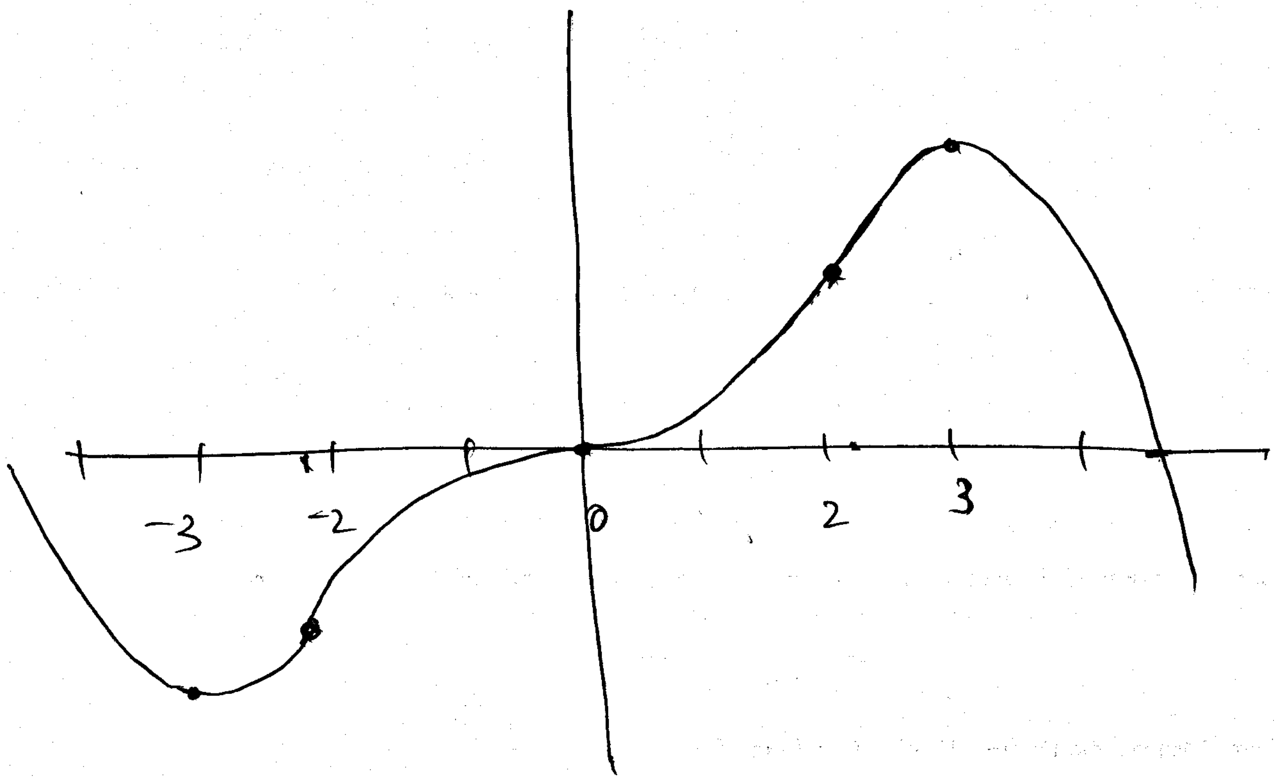
$\approx 2.1 \qquad \approx -2.1$



$f''(-3) = (-)(-) > 0 \qquad f''(-1) = (-)(+) < 0$

$f''(1) = (+)(+) > 0 \qquad f''(3) = (+)(-) < 0$

$t = 0, \frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}}$ inflection points



$f(-3) = 15(-27) - 9(-27) = (15-9)(-27) = -162$

$f(0) = 0, \quad f(3) = 162$

<u>e.g.</u> $f(x) = x^{2/3}(x^2-4)$

$f'(x) = \frac{8}{3}\left(\frac{x^2-1}{x^{1/3}}\right) =$

CP: $x = 0$, $x = 1$, $x = -1$

f' undef $\qquad f' = 0$

DECR o INCR • DECR o INCR

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

local max
since $f(0)$ exists
(even though $f'(0)$ doesn't)

$f''(x) = \frac{8}{3}\left(\frac{x^{1/3} \cdot 2x - (x^2-1)\frac{1}{3}x^{-2/3}}{x^{2/3}}\right)$

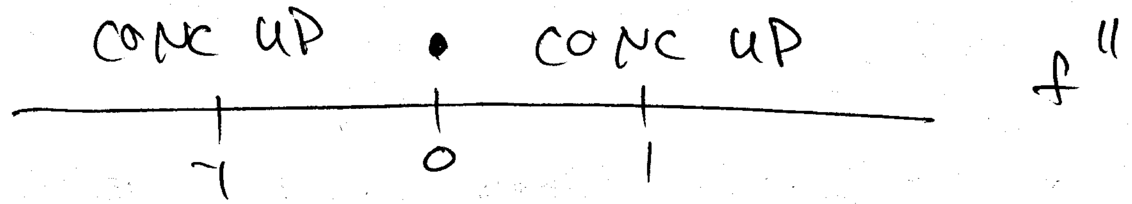
$= \frac{8}{3}\left(\frac{2x^{4/3} - \frac{1}{3}x^{4/3} + \frac{1}{3}x^{-2/3}}{x^{2/3}}\right)$

$= \frac{8}{3}\left(\frac{5}{3}x^{2/3} + \frac{1}{3}x^{-4/3}\right)$

$= \frac{8}{9}\left(5x^{2/3} + \frac{1}{x^{4/3}}\right)$

$= \frac{8}{9}\left(\frac{5x^2+1}{x^{4/3}}\right)$

Possible inflection pts: $x=0$.



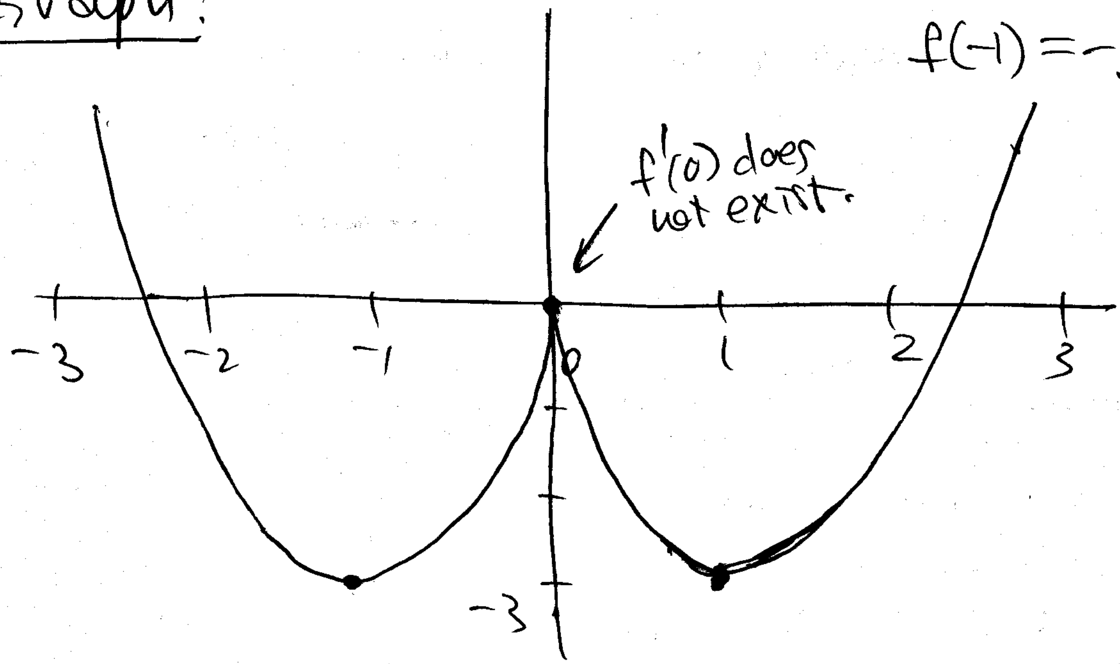
$$f''(-1) = \frac{(+)}{(+)} > 0 \qquad f''(1) = \frac{(+)}{(+)} > 0$$

So <u>NO INFLECTION PTS</u>.

<u>Graph:</u>

$f(0)=0 \quad f(1)=-3$

$f(-1)=-3$



4.3 Graphing Functions

e.g. $f(x) = \dfrac{x^2}{x^2-4}$

$$f'(x) = \frac{(x^2-4)(2x)-(x^2)(2x)}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$$

$$f''(x) = \frac{(x^2-4)^{\cancel{2}}(-8)-(-8x)(2\cancel{(x^2-4)}(2x))}{(x^2-4)^{\cancel{4}\,3}}$$

$$= \frac{-8x^2+32+32x^2}{(x^2-4)^3}$$

$$= \frac{24x^2+32}{(x^2-4)^3}$$

Incr/decr: CP: $x=0$, $x=2$, $x=-2$

INCR • INCR ∘ DECR • DECR.

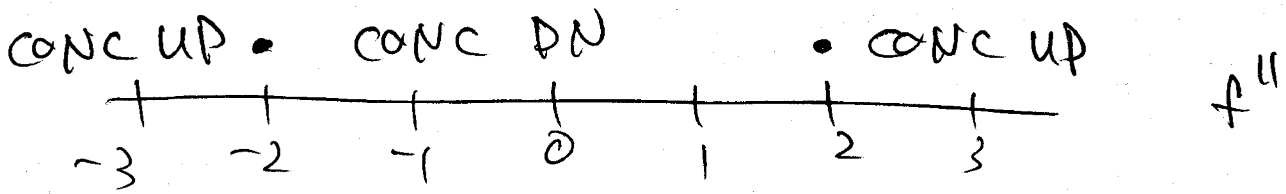
$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

$f'(-3) = \dfrac{(+)}{(+)} > 0$ $\quad f'(-1) = \dfrac{(+)}{(+)}$ $\quad f'(1) = \dfrac{(-)}{(+)} < 0$

$f'(3) = \dfrac{(-)}{(+)} < 0$ $\quad \boxed{x=0 \text{ local max}}$

concave up/down:

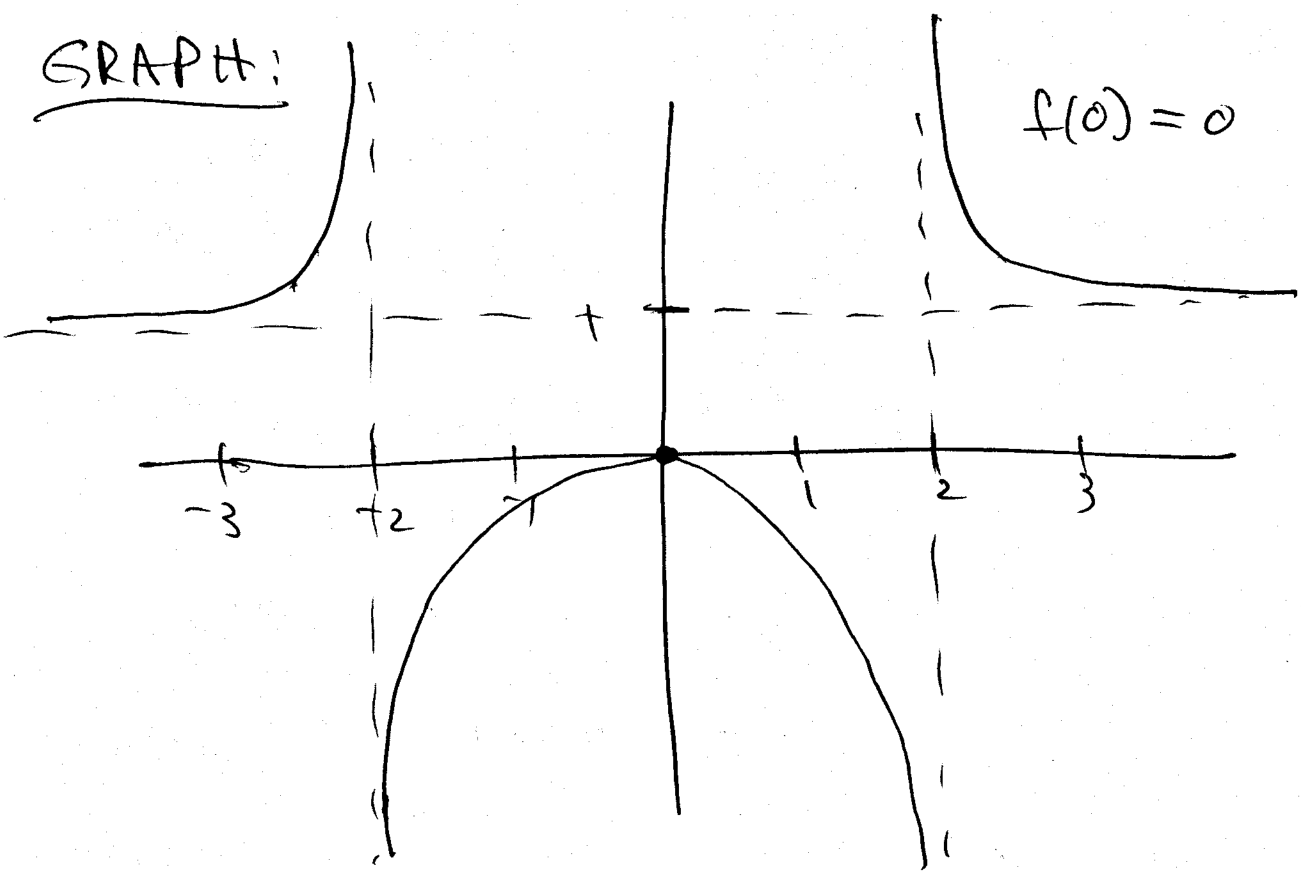
Possible inflection pts: $x=2, x=-2$



$f''(-3) = \frac{(+)}{(+)} > 0 \quad f''(0) = \frac{(+)}{(-)} < 0 \quad f''(3) = \frac{(+)}{(+)} > 0$

GRAPH:

$f(0) = 0$



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