Quiz 7 – 3.6, 3.7

Finding maxima/minima of functions
– 2 kinds: Absolute extrema
Local extrema.

– Thm: If f is continuous on $I = [a,b]$ then f has an absolute max and an absolute min.

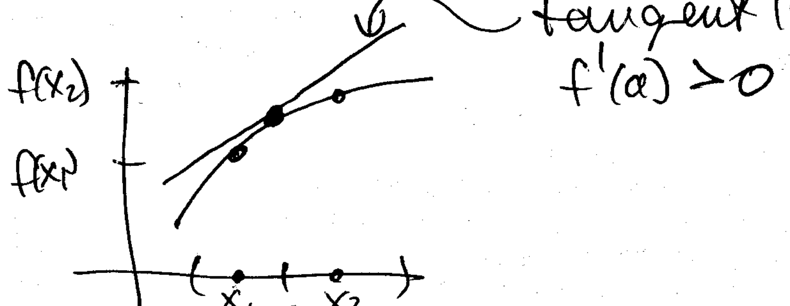
– Thm: If f is differentiable on (a,b) and continuous on $[a,b]$ then the absolute extrema occur when
① f has a ~~end~~ critical point ($f' = 0$)
② at the end points.

– Thm: All local extrema of a function f occur when f has a critical point, i.e.
~~f has a critical po~~
when:
① $f' = 0$ or
② f' is undefined.

4.2 What Derivatives Tell Us.

A. First derivative

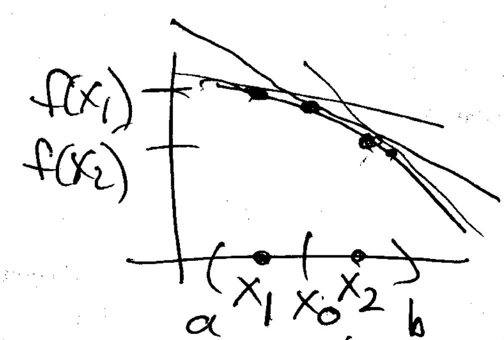
$f'(x)$ = slope of the tangent line



Idea: for x near a the graph of $f(x)$ follows the tangent line.

Conclusion: (a) If $f'(x) > 0$ for each x in (a, b) then $f(x)$ is increasing, i.e., if $x_1 < x_2$ then $f(x_1) < f(x_2)$

(b) If $f'(x) < 0$ for each x in (a, b) then $f(x)$ is decreasing, i.e. if $x_1 < x_2$ then $f(x_1) > f(x_2)$.



Q: Where have we seen this already?

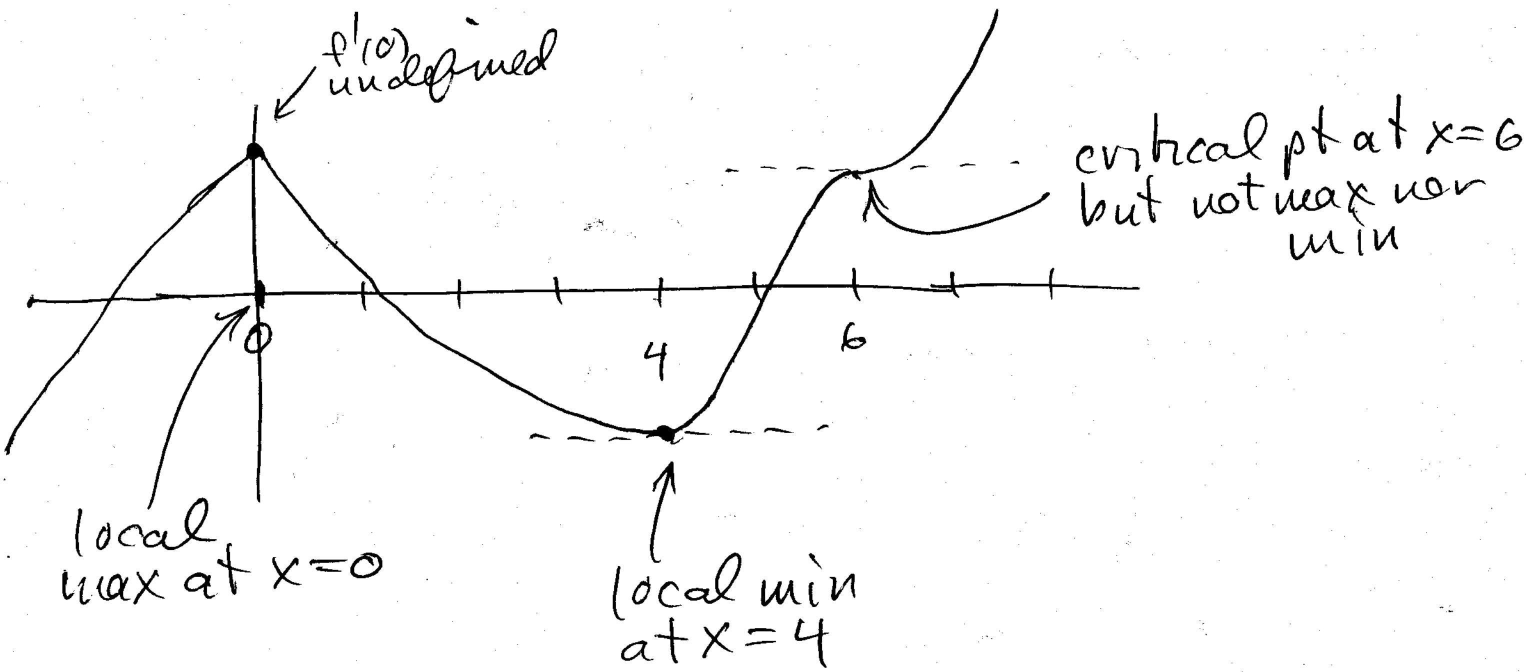
$s(t)$ – position at time t.

$v(t) = s'(t)$ $\quad s'(t) > 0$ moving to right, i.e. s is increasing.

$s'(t) < 0$ moving left, i.e. s is decreasing.

e.g. 1

1. $f' > 0$ on $(-\infty, 0), (4, 6), (6, \infty)$
2. $f' < 0$ on $(0, 4)$
3. $f'(0)$ undefined
4. $f'(4) = f'(6) = 0$



e.g. $g(x) = x^4 - 4x^3 + 4x^2$

Find intervals of increase/decrease for g.

$g'(x) = 4x^3 - 12x^2 + 8x$

① Find all critical points.

$4x^3 - 12x^2 + 8x = 0$

$4x(x^2 - 3x + 2) = 0$

$4x(x-1)(x-2) = 0$

$x = 0, x = 1, x = 2 \leftarrow$ crit. pt.

② Identify intervals of incr/decr.

	g decreasing		g incr.		g decr		g incr.	
	$g'<0$	0	$g'>0$	0	$g'<0$	0	$g'>0$	$g'(x)$
-1		0		1		2		3

$$g'(-1) = -4-12-8 < 0$$

$$g'\left(\tfrac{1}{2}\right) = \frac{4}{8} - 3 + 4 = \frac{1}{2} + 1 > 0$$

$$g'\left(\tfrac{3}{2}\right) = 4\cdot\frac{27}{8} - 12\cdot\frac{9}{4} + 8\cdot\frac{3}{2} = \frac{27}{2} - 27 + 12 = -\frac{3}{2} < 0$$

$$g'(3) = 4\cdot 27 - 12\cdot 9 + 8\cdot 3 = 108 - 108 + 24 > 0.$$

g increasing on $(0,1)$, $(2,\infty)$

g decreasing on $(-\infty,0)$, $(1,2)$.

e.g. $f(t) = 15t^3 - t^5$

Find intervals of incr/decr of f.

$$f'(t) = 45t^2 - 5t^4$$

$$45t^2 - 5t^4 = 0$$

$$5t^2(9-t^2) = 0$$

$$5t^2(3-t)(3+t) = 0$$

$t=0,\ t=3,\ t=-3 \leftarrow$ crit pts

$f'<0$ DECR $\quad 0 \quad$ $f'>0$ INCR $\quad 0 \quad$ $f'>0$ INCR $\quad 0 \quad$ $f'<0$ DECR

$f'(x)$: $-4 \quad -3 \quad 0 \quad 3 \quad 4$

$$f'(-4) = (+)(+)(-) < 0$$
$$f'(-1) = (+)(+)(+) > 0$$
$$f'(1) = (+)(+)(+) > 0$$
$$f'(4) = (+)(-)(+) < 0$$

$$\frac{2}{3} - 1 = \frac{2}{3} - \frac{3}{3} = -\frac{1}{3}$$

e.g. $h(x) = x^{2/3}(x^2-4)$

$$h'(x) = x^{2/3}(2x) + (x^2-4)\left(\frac{2}{3}x^{-1/3}\right)$$
$$= 2x^{5/3} + \frac{2}{3}x^{5/3} - \frac{8}{3}x^{-1/3}$$
$$= \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3}$$
$$= \frac{8}{3}\left(x^{5/3} - \frac{1}{x^{1/3}}\right) = \frac{8}{3}\left(\frac{x^2-1}{x^{1/3}}\right)$$

$h'(0)$ undefined $\quad$ $x = 0$

$x^2 - 1 = 0$

$x = 1$, $x = -1$ $\quad$ critical points

DECR o INCR • DECR o INCR

$-2 \quad -1 \quad 0 \quad 1 \quad 2 \qquad h'(x)$

$h'(-2) = \dfrac{(+)}{(-)} < 0$

$h'(\tfrac{1}{2}) = \dfrac{(-)}{(+)} < 0$

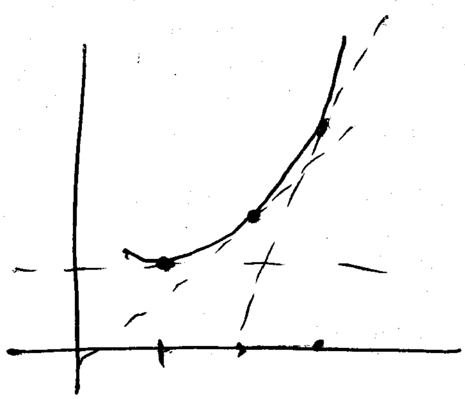
$h'(-\tfrac{1}{2}) = \dfrac{(-)}{(-)} > 0$

$h'(2) = \dfrac{(+)}{(+)} > 0$

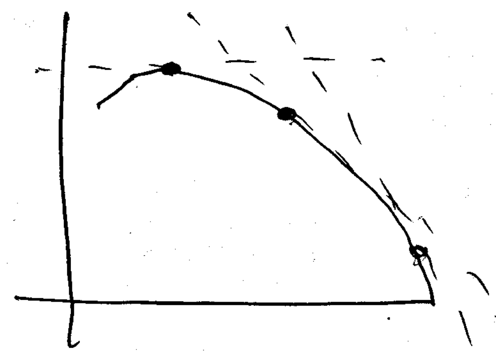
B. Second Derivative

Idea: $f''(x)$ is the derivative of $f'(x)$

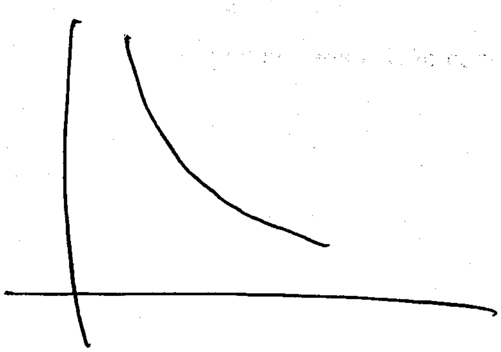
So $f''(x)$ tells us whether $f'(x)$ is increasing or decreasing.



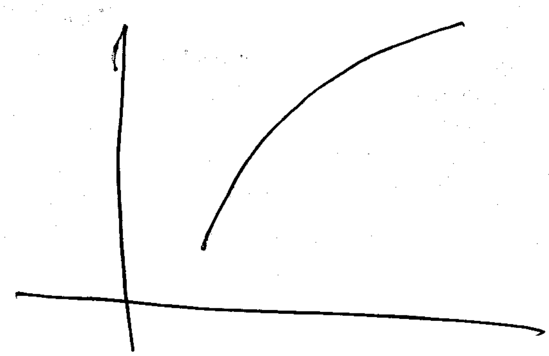
f' increasing
$f'' > 0$
concave up



f' decreasing
$f'' < 0$
concave down.



f decreasing
f conc. up.



f increasing
f conc down