$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

① slope of tangent line

② instantaneous rate of change of y with respect to x $(y = f(x))$

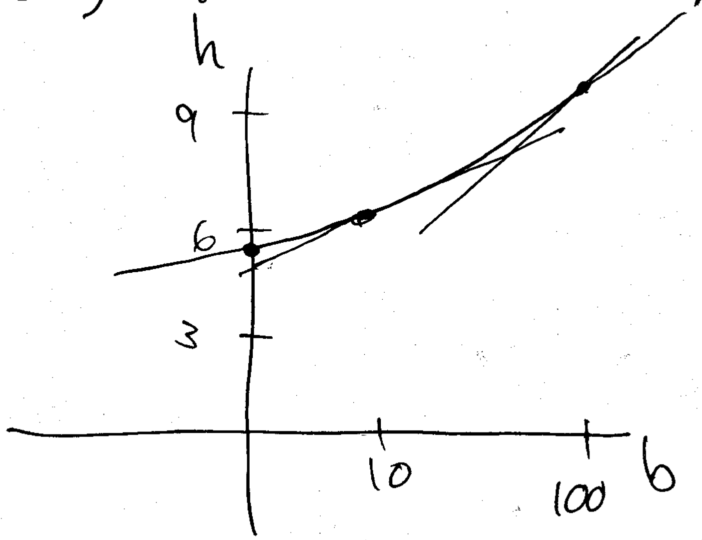
Basic example: $s = s(t)$ (position at time t)

$s'(t) =$ velocity

$|s'(t)| =$ speed

$s''(t) =$ acceleration

#38) $h = 5.67 + 0.7b + .0067b^2$



$\frac{dh}{db} = 0.7 + .0134\,b$ in $\frac{\text{meters}}{\text{centimeter}}$

Say $b = 10$ cm

$$\left.\frac{dh}{db}\right|_{b=10} = 0.7 + .134$$

$$= .834 \frac{m}{cm}$$

Because $\frac{dh}{db} \approx \frac{\Delta h}{\Delta b}$ so $\Delta h \approx \left(\frac{dh}{db}\right) \Delta b$

say $b = 100$ cm

$$\left.\frac{dh}{db}\right|_{b=100} = 0.7 + 1.34 = 2.04 \ \frac{m}{cm}$$

Marginal Cost.

$C(x)$ = cost (in \$) of producing x units of some commodity

$C'(x)$ = marginal cost.

$$= \frac{\text{dollars}}{\text{unit}}$$

Say $C'(500) = 25$

Means: cost of producing 501st unit is about \$25.

$$C'(x) = \frac{dC}{dx} \approx \frac{\Delta C}{\Delta x} \rightarrow \Delta C \cong \left(\frac{dC}{dx}\right)\Delta x$$

3.6 Chain Rule

e.g. $y = (3x^2+5)^{1/2}$, $y = e^{x^2}$

Our rules do not cover either of these.

Idea: Each of these functions is a composite, i.e. $y = f(g(x)) = (f \circ g)(x)$

$y = (3x^2+5)^{1/2}$ $\quad g(x) = 3x^2+5 \quad f(u) = u^{1/2}$

$y = e^{x^2}$ $\quad \underbrace{g(x) = x^2}_{\text{inner}} \quad \underbrace{f(u) = e^u}_{\text{outer}}$

How to take derivative:

Need this idea: $\dfrac{dy}{dx} \approx \dfrac{\Delta y}{\Delta x}$ $\quad \Delta y \approx \left(\dfrac{dy}{dx}\right)\Delta x$

think Δx "magnified" by this number.

Look at $y = f(g(x))$

Want $\dfrac{dy}{dx}$. Ask: Given small change dx, what is corresp. change dy?

Two steps: $u = g(x) \quad y = f(u)$

① dx induces change in u, $du = \left(\frac{dy}{dx}\right) dx$

② du induces change in y, $dy = \left(\frac{dy}{du}\right) du$

So $$dy = \left(\frac{dy}{du}\right) du = \left(\frac{dy}{du}\right)\left(\frac{dy}{dx}\right) dx$$

$$f'(u) \quad g'(x)$$

$$\boxed{\begin{array}{l} \dfrac{dy}{dx} = f'(u)g'(x) = f'(g(x)) \cdot g'(x) \\ \underline{\text{or}} \quad \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} \end{array}}$$

<u>e.g.</u> $y = (3x^2+5)^{1/2}$ $\quad f(u) = u^{1/2}$, $g(x) = \underline{3x^2+5}$

$f'(u) = \frac{1}{2}u^{-1/2}$, $g'(x) = 6x$

$$\frac{dy}{dx} = \left(\frac{1}{2}u^{-1/2}\right)(6x) = \frac{1}{\cancel{2}}(3x^2+5)^{-1/2}(\cancel{6}^{3}x)$$

$$= 3x(3x^2+5)^{-1/2}.$$

e.g. $y = e^{x^2}$ $\quad f(u) = e^u \quad g(x) = x^2$

$f'(u) = e^u \quad g'(x) = 2x$

$$\frac{dy}{dx} = f'(\underbrace{g(x)}_{u}) \cdot g'(x) = e^{x^2} \cdot 2x = \underline{2xe^{x^2}}$$

Two new rules:

① $\frac{d}{dx}[g(x)^n] = n[g(x)]^{n-1} \cdot g'(x)$

$f(u) = u^n$
$f'(u) = nu^{n-1}$

② $\frac{d}{dx} e^{g(x)} = e^{g(x)} \cdot g'(x)$

$f(u) = e^u$
$f'(u) = e^u$

e.g. $y = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-1/2}(2x) = x(x^2+1)^{-1/2}$$

eg $y = \tan(5x^2)$ $\boxed{\begin{array}{l} f(u) = \tan(u) \\ g(x) = 5x^2 \end{array}}$

$$\frac{dy}{dx} = \sec^2(5x^2) \cdot 10x$$
$$= 10x \sec^2(5x^2)$$

e.g. $y = \cos^4\theta + \sin^4\theta = (\cos\theta)^4 + (\sin\theta)^4$
$(\neq \cos(\theta^4) + \sin(\theta^4))$

$$\frac{dy}{d\theta} = 4(\cos\theta)^3(-\sin\theta) + 4(\sin\theta)^3(\cos\theta)$$
$$= 4(-\sin\theta\cos^3\theta + \cos\theta\sin^3\theta)$$

e.g. $y = \sin^2 x^2 = (\sin(x^2))^2$

$$\frac{dy}{dx} = 2\sin(x^2) \cdot \frac{d}{dx}\sin(x^2)$$
$$= 2\sin(x^2) \cdot \cos(x^2) \cdot 2x$$
$$= 4x\sin(x^2)\cos(x^2)$$