3.5 Derivatives as Rates of Change.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Interpretation:

① Slope of tangent line to graph of f at x.

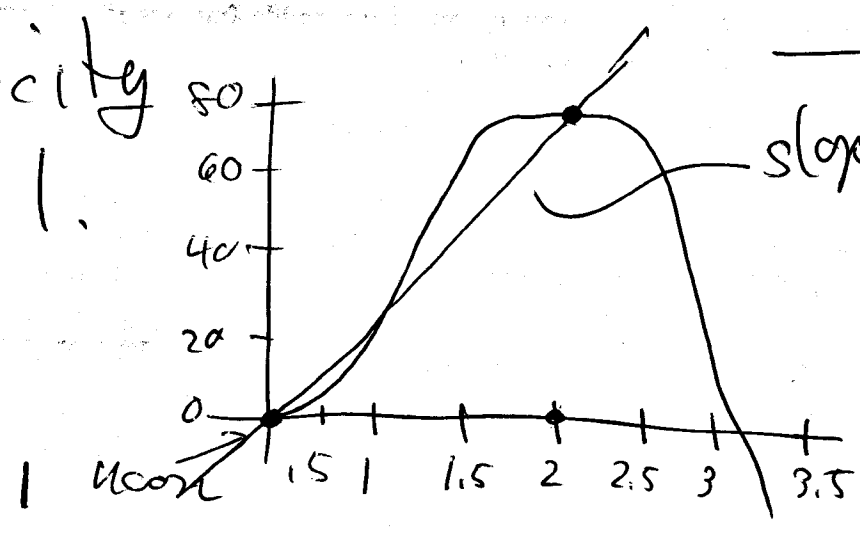
② If we let $y = f(x)$ then $f'(x)$ is the instantaneous rate of change of y with respect to x.

e.g. if y = position of object at time x

x = time.

Then $\frac{dy}{dx}$ = velocity (in $\frac{mi}{hr}$, $\frac{ft}{sec}$, $\frac{m}{sec}$ etc...)

e.g. Velocity Example 1.



b. Avg velocity betw noon and 2 PM

$t=0$ $t=2$.

$$V_{avg} = \frac{(\text{pos at } t=2) - (\text{pos at } t=0)}{2-0} = \frac{\Delta s}{\Delta t}$$

change in position; change in time

$$= \frac{80}{2} = 40 \frac{mi}{hr}.$$

c. $$V_{avg} = \frac{\Delta s}{\Delta t} = \frac{(-20) - (80)}{3.5 - 2} = \frac{-100}{1.5}$$

$$\approx -67 \frac{mi}{hr}.$$

Q: Avg velocity between 12:30 ($t = \frac{1}{2}$) and 3 pm ($t=3$)?

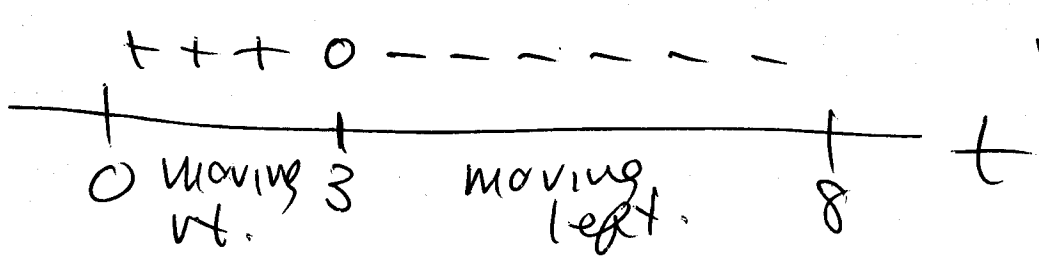
$$V_{avg} = \frac{\Delta s}{\Delta t} = \frac{20-20}{3-0.5} = 0 \frac{mi}{hr}.$$

Avg velocity: $\frac{\Delta s}{\Delta t}$ ← change in pos / ← change in time

Inst velocity: $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

eg #14) $f(t) = 18t - 3t^2 \quad 0 \le t \le 8$

Velocity: $v(t) = f'(t) = 18 - 6t$



Stationary: $v(t) = 0$

$18 - 6t = 0 \quad t = 3$

Moving right: $v(t) > 0$

$v(0) = 18 > 0 \quad v(1) = 12 > 0.$ ← check some values of t

Moving left: $v(t) < 0$

$v(6) = 18 - 36 < 0.$ ←

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acceleration: $a(t) = v'(t) = f''(t) = -6$

$v(1) = 18 - 6 = 12 \ \frac{ft}{sec}$

$a(1) = -6 \ ft/sec^2$ ← means velocity is decreasing from 12 toward 0

so speed ($= |v(t)|$) is decreasing

so SLOWING DOWN.

Say $t = 5$.

$v(5) = 18 - 6 \cdot 5 = -12$ ft/sec.

Speed $= |v(5)| = 12$ ft/sec.

$a(5) = -6$ ft/sec^2

Velocity is becoming more negative

so speed $(= |v(t)|)$ is <u>increasing</u>

so SPEEDING UP.