Quiz 4 – 2.7, 3.1

Exam 1 – Wed 9-26 (see web page)

Recap:

Derivative → $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x)$ = slope of tangent line to graph of f at x.

Power rule → $f(x) = x^n$, $f'(x) = nx^{n-1}$ for $n = 0, 1, 2, 3, \ldots$

Derivative is "linear" → Derivatives distributes across sums and multiplication by constants

$(f+g)'(x) = f'(x) + g'(x)$ $\quad$ $(cf)'(x) = cf'(x)$

eg. #10) $g(x) = e^3$ $\quad$ $g'(x) = 0$

$f(v) = v^{100}$ $\quad$ $f'(v) = 100v^{99}$

$g(w) = \frac{5}{6}w^{12}$ $\quad$ $g'(w) = \frac{5}{6}(12w^{11}) = 10w^{11}$

$h(x) = (x+1)(x^5+6x) = x^6+6x^2+x^5+6x$

$h'(x) = 6x^5+12x+5x^4+6$

$6x^1 \to 6x^0$

$f(t) = 6\sqrt{t} - 4t^2 + 9$

$f'(t) = 6\left(\frac{d}{dt} t^{1/2}\right) - 8t + 0$

$$= \lim_{h\to 0} \frac{(t+h)^{1/2} - t^{1/2}}{h} \cdot \frac{(t+h)^{1/2}+t^{1/2}}{(t+h)^{1/2}+t^{1/2}}$$

$$= \lim_{h\to 0} \frac{\cancel{t}+h-\cancel{t}}{h((t+h)^{1/2}+t^{1/2})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}((t+h)^{1/2}+t^{1/2})}$$

$$= \lim_{h\to 0} \frac{1}{(t+h)^{1/2}+t^{1/2}} = \frac{1}{2t^{1/2}} = \boxed{\frac{1}{2}t^{-1/2}}$$

$= 6\left(\frac{1}{2}t^{-1/2}\right) - 8t$

$= 3t^{-1/2} - 8t$ //

e.g. Find equ of tangent line to

$f(x) = 3x^4 + 5x^3 + 2$ at $a = 1$

point: $(1, f(1)) = (1, 10)$

slope: $f'(x) = 12x^3 + 15x^2 \quad f'(1) = 27$

Equ of line: $y - 10 = 27(x-1)$

$y = 27x - 17$ //

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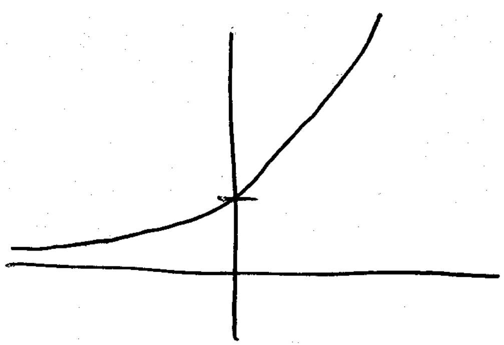
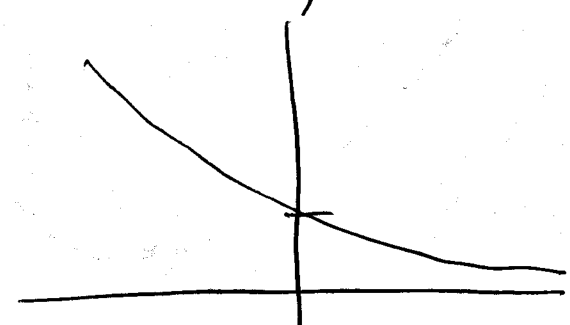
② Derivative of $f(x) = a^x$, $a > 0$.



$a > 1$



$0 < a < 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

$$\frac{a^x(a^h - 1)}{h}$$

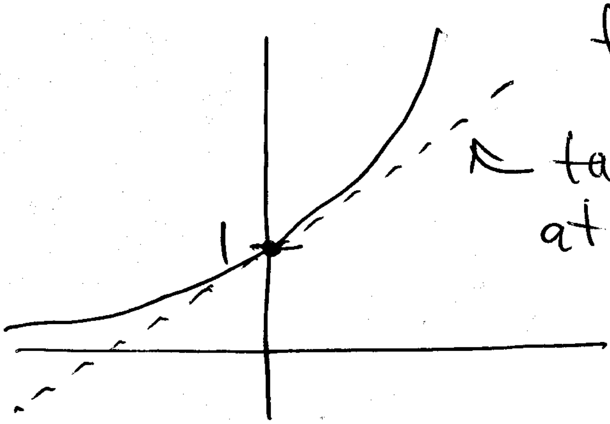
$$= \lim_{h \to 0} \frac{a^x a^h - a^x}{h} = \lim_{h \to 0} a^x \left(\frac{a^h - 1}{h} \right)$$

$$= a^x \underbrace{\left(\lim_{h \to 0} \frac{a^h - 1}{h} \right)}_{\text{just a number.}}$$

Know: $f(x) = a^x \quad f'(x) = (\text{const})\, a^x$

What is (const)?

$f(x) = a^x$



tang line at $x=0$ $\Big\}$ slope $= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{h \to 0} \frac{a^h - 1}{h} = (\text{const})$$

$\underline{a = 2}$

h	$\frac{a^h - 1}{h}$
.1	.718
.01	.696
.001	.693

$\underline{a = 3}$

h	$\frac{a^h - 1}{h}$
.1	1.161
.01	1.105
.001	1.099

<u>Q:</u> Somewhere between $a=2$ and $a=3$ is a number so that $\lim_{h \to 0} \frac{a^h - 1}{h} = 1$

<u>A:</u> $a = e \approx 2.71828\ldots$

$$\frac{a^h - 1}{h} \approx 1 \longrightarrow a^h - 1 \approx h \rightarrow a^h \approx 1 + h$$

$$\longrightarrow a \approx (1+h)^{1/h}$$

$$a = \lim_{h \to 0} (1+h)^{1/h}$$

h	$(1+h)^{1/h}$
.1	2.594
.01	2.705
.001	2.717

$$f(x) = e^x \qquad f'(x) = e^x$$

No!

$$f(x) = e^x \longrightarrow \cancel{f'(x) = xe^{x-1}}$$

No! No!

eg $g(x) = 3x^2 + 4e^x$

$g'(x) = 6x + 4e^x$

RSHI 301
or 302

3.3 Product Rule + Quotient rule

Ⓐ Product Rule.

$f(x) = u(x)v(x)$

$f(x) = c \cdot v(x)$ $\quad f'(x) = c \cdot v'(x)$

$f(x) = u(x) \cdot v(x)$ $\quad f'(x) \neq u(x)v'(x)$ No!

$f'(x) \neq u'(x)v'(x)$ No!
No!

$f(x) = u(x) \cdot v(x)$ $\quad \boxed{f'(x) = u(x)v'(x) + v(x)u'(x)}$

Idea $\lim_{h \to 0} \frac{u(x+h)v(x+h) - u(x)v(x)}{h}$ is kind of involved.

eg $h(x)=(x+1)(x^5+6x)$

Product rule:

$$h'(x)=(x+1)(5x^4+6)+(x^5+6x)(1)$$
$$=5x^5+6x+5x^4+6+x^5+6x$$
$$=6x^5+5x^4+12x+6.$$

Note: $h'(x)\neq(1)(5x^4+6)$ No Good

e.g. $f(x)=x^2e^x$

$$f'(x)=(x^2)(e^x)+(e^x)(2x)$$
$$=x^2e^x+2xe^x$$
$$=xe^x(x+2)$$

Notation and Higher Derivatives

$$f(x) \longrightarrow f'(x)$$

$$\longrightarrow \frac{d}{dx}(f)$$

$$\longrightarrow \frac{df}{dx}$$

$$\longrightarrow \frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$y = f(x)$$

$$f(x) \longrightarrow f'(x) \longrightarrow f''(x) \longrightarrow f'''(x) \longrightarrow f^{(4)}(x) \longrightarrow \cdots$$

$$\longrightarrow \frac{d}{dx}(f) \longrightarrow \frac{d}{dx}\left(\frac{d}{dx}(f)\right) = \frac{d^2}{dx^2}(f) \longrightarrow \frac{d^3}{dx^3}(f)$$

$$\longrightarrow \frac{df}{dx} \longrightarrow \frac{d^2f}{dx^2} \longrightarrow \frac{d^3f}{dx^3} \longrightarrow \cdots$$

$$\frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2} \longrightarrow \frac{d^3y}{dy^3} \longrightarrow \cdots$$

eg: $h(x) = (x+1)(x^5+6x)$

$$h'(x) = (x+1)(5x^4+6) + (x^5+6x)$$

$$h''(x) = (x+1)(20x^3) \cancel{+ (5x^4+6)}$$

$$\cancel{h''(x) =} \quad + (5x^4+6)(1) + (5x^4+6)$$

$$= 20x^3(x+1) + 2(5x^4+6)$$

$$h'''(x) = 20x^3(1) + (x+1)(60x^2) + 2(20x^3)$$

$$= 20x^3 + 60x^2(x+1) + 40x^3$$

$$= 60x^3 + 60x^2(x+1)$$