Quiz 2 2.3, 2.4

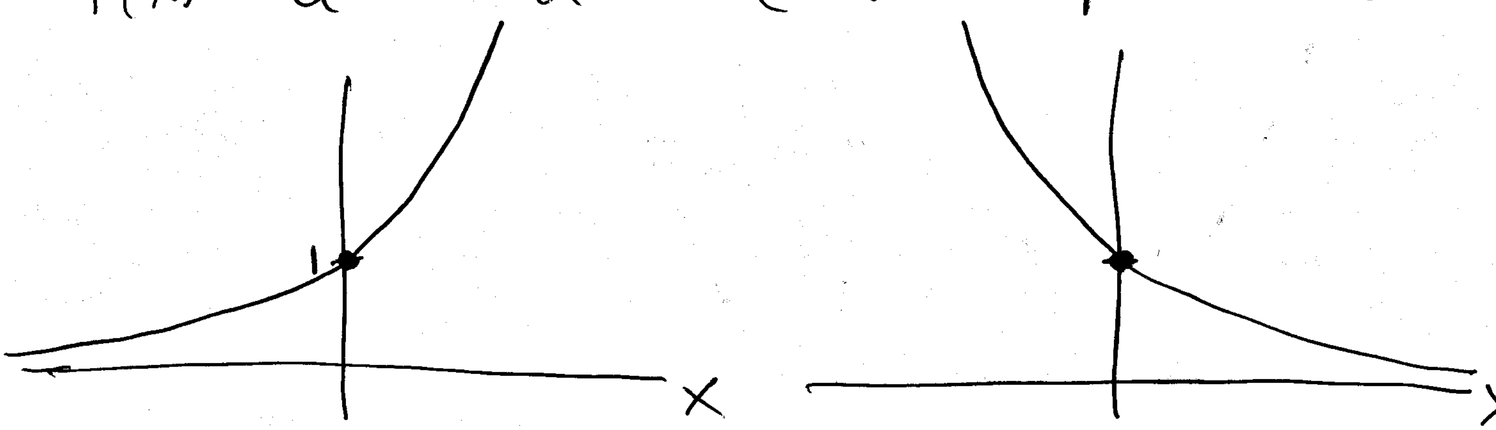
$\underbrace{\text{Infinite Limits}}$ and $\underbrace{\text{Limits at infinity}}$

$$\lim_{x \to a^{\pm}} f(x) = \pm\infty. \qquad \lim_{x \to \pm\infty} f(x) = L$$

$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

Exponentials

$f(x) = a^x$ $a > 0$ (base of exponential)



$a > 1$
(say e.g. $a = 2$, $f(x) = 2^x$)

$0 < a < 1$
e.g. $f(x) = \left(\frac{1}{3}\right)^x = 3^{-x}$
so $\left(\frac{1}{3}\right)^x$ is the reflection of $g(x) = 3^x$ through y-axis.

$\left(\frac{1}{3}\right)^x = f(x) = g(-x)$ $g(x) = 3^x$

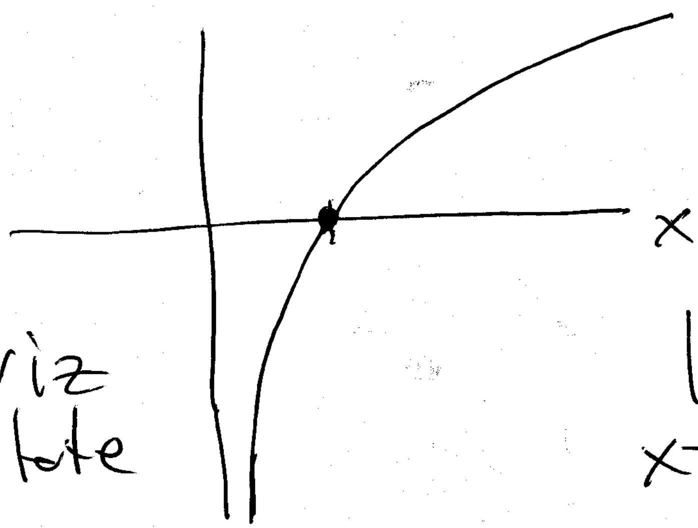
If $a>1$ $\lim_{x\to-\infty} a^x = 0$ $\lim_{x\to\infty} a^x = \infty$

horiz asymp at $y=0$
no vertical asymp.

If $0<a<1$ $\lim_{x\to\infty} a^x = 0$ $\lim_{x\to-\infty} a^x = \infty$

horiz asy at $y=0$, no vertical asymp.

eg $f(x) = \ln(x)$. $\ln(1) = 0$



vertical asymp at $x=0$

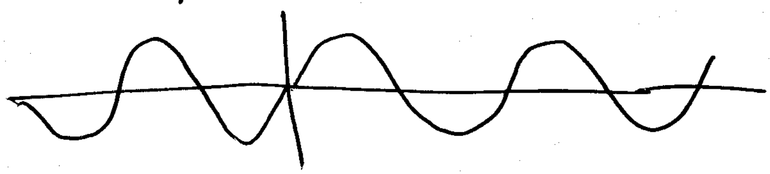
$\lim_{x\to0^+} \ln(x) = -\infty$

no horiz asymptote

Trig functions

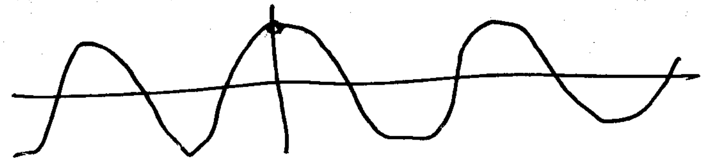
vertical asymptotes.

sin(x)

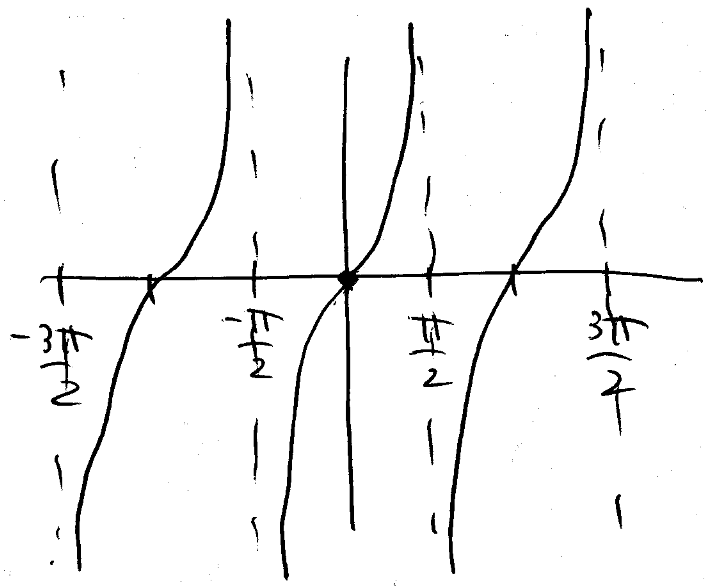


no asymptotes

cos(x)



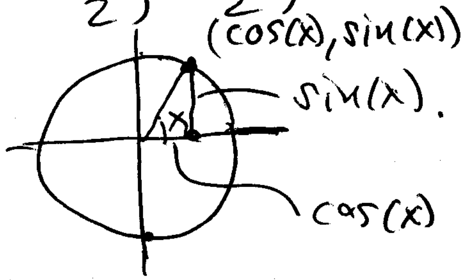
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

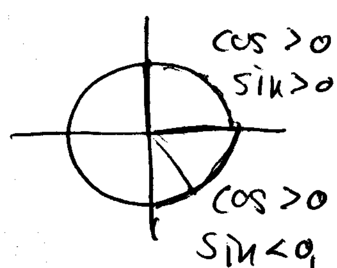


$\cos(x) = 0$

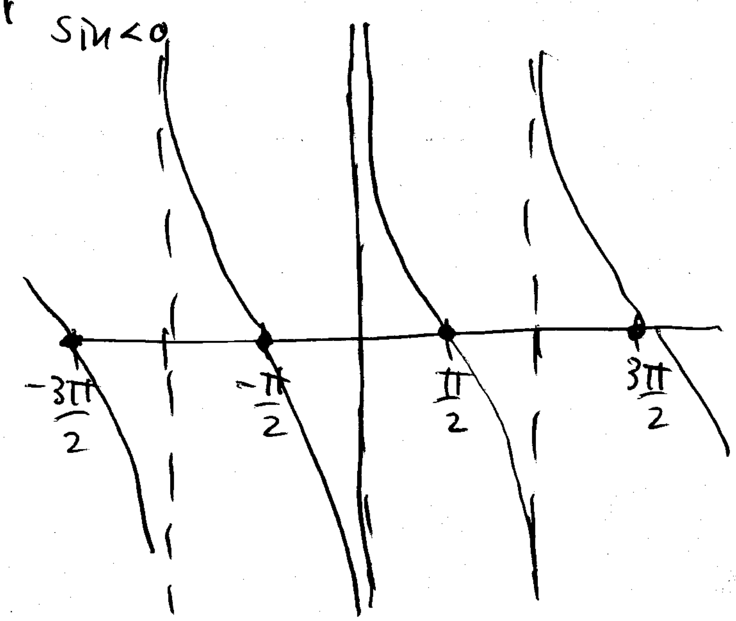
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, \ldots$

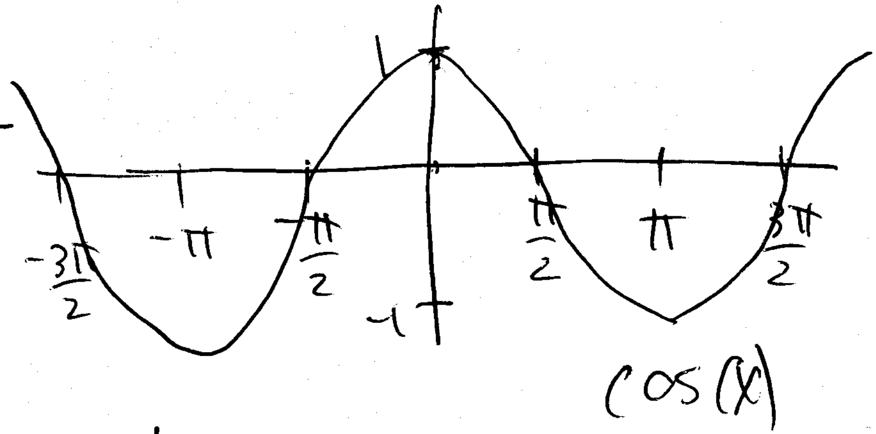


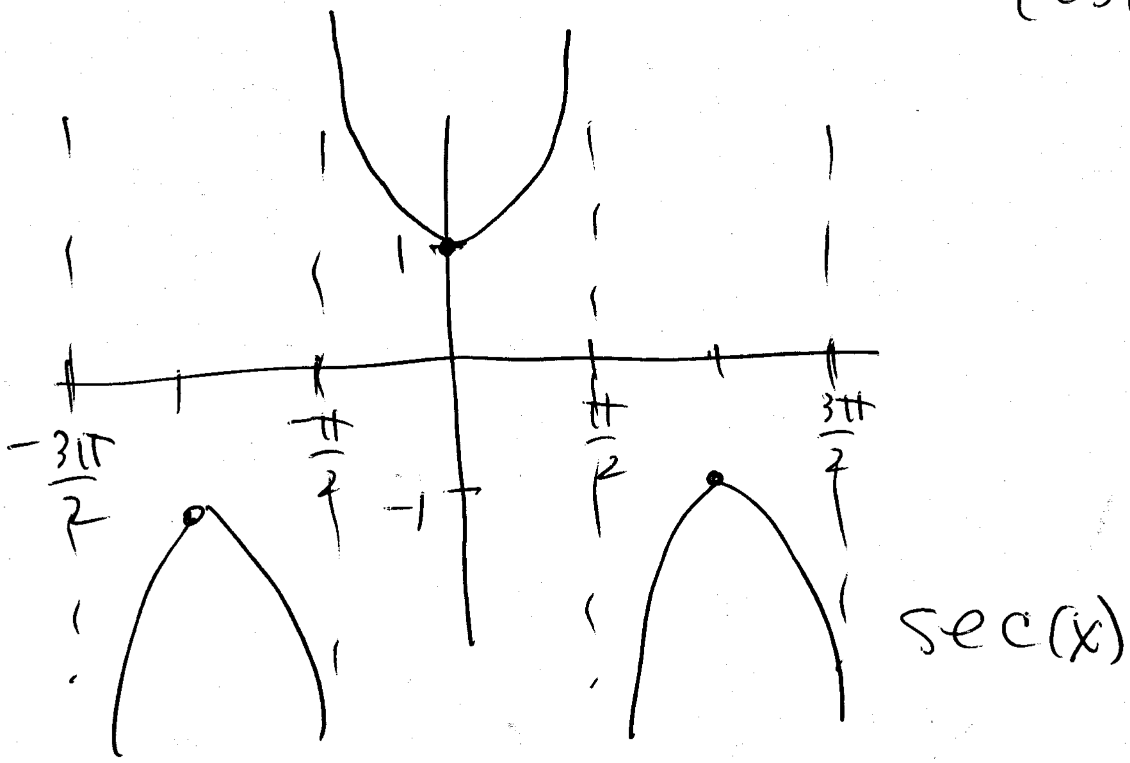


$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$



$$\sec(x) = \frac{1}{\cos(x)}$$





2.6 Continuity

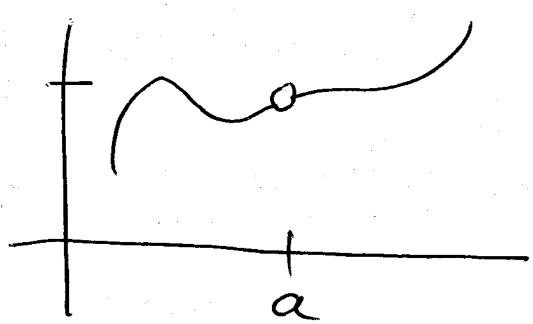
$$\lim_{x \to a} f(x) = f(a)$$ f is continuous at a if

Three things must happen:

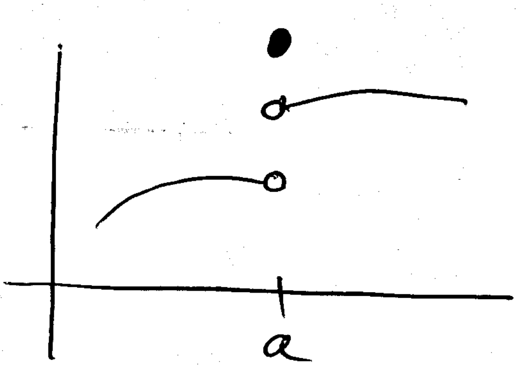
① $f(a)$ must exist.

② $\lim_{x \to a} f(x)$ must exist.

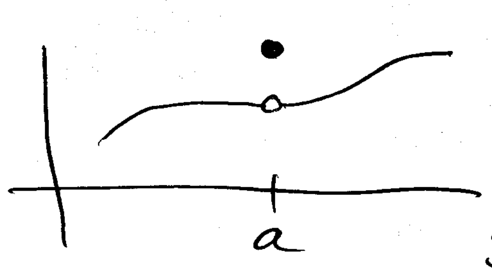
③ both must be equal.



① fails
② holds
f not continuous at a.



① holds.
② fails.
f is not cont. at a.



① holds ② holds
but ③ fails.
f not cont at a.

#10) Points of discont:

$x=1$ ① holds, ② holds, ③ fails

$x=2$ ① holds, ② fails

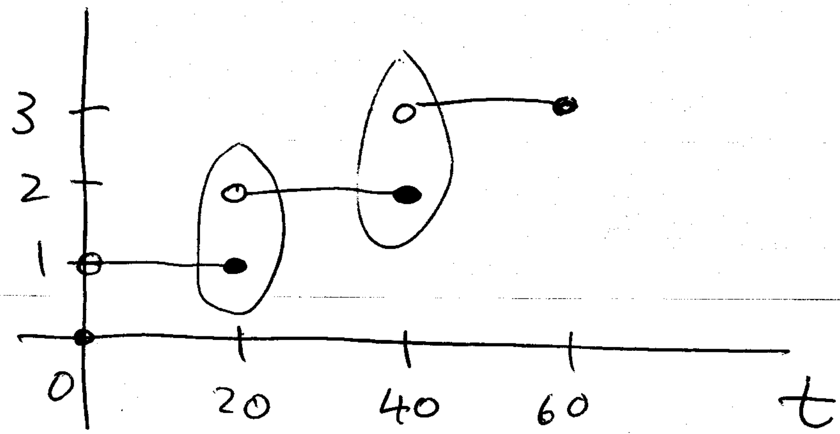
$x=3$ ① fails, ② holds.

$\lim_{x\to 0^+} f(x) = f(0)$ so f is cont from the right at $x=0$

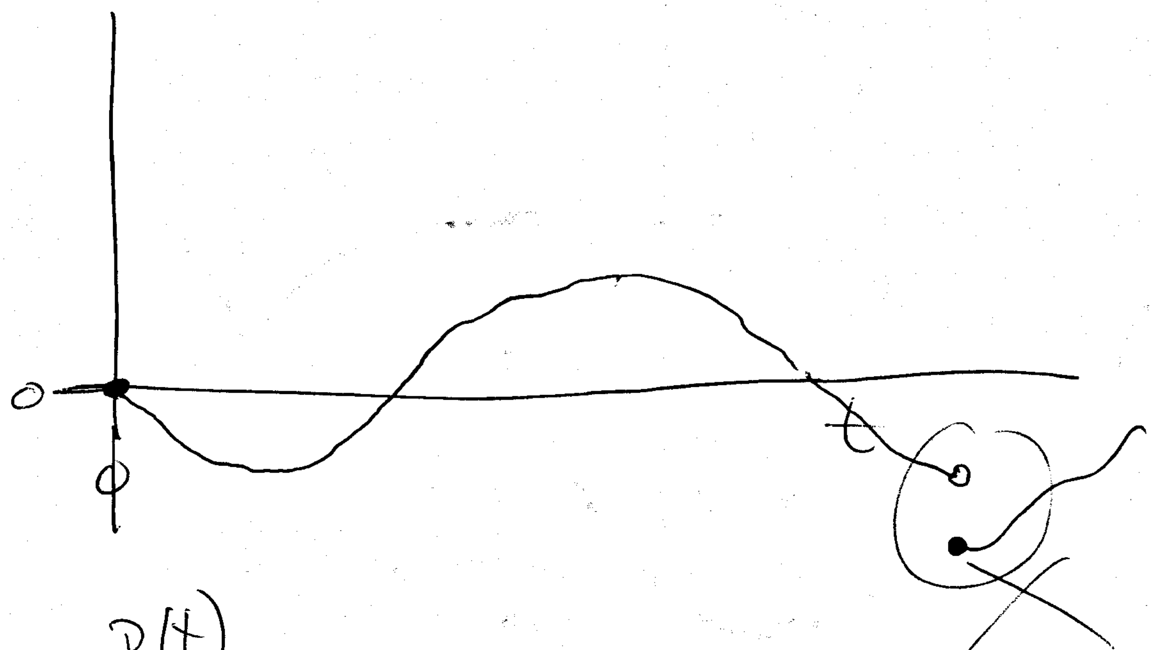
$\lim_{x\to 4^-} f(x) = f(4)$ so f cont from the left at $x=4$.

Say f is continuous on $[0,1)\cup(1,2)\cup(2,3)\cup(3,4]$

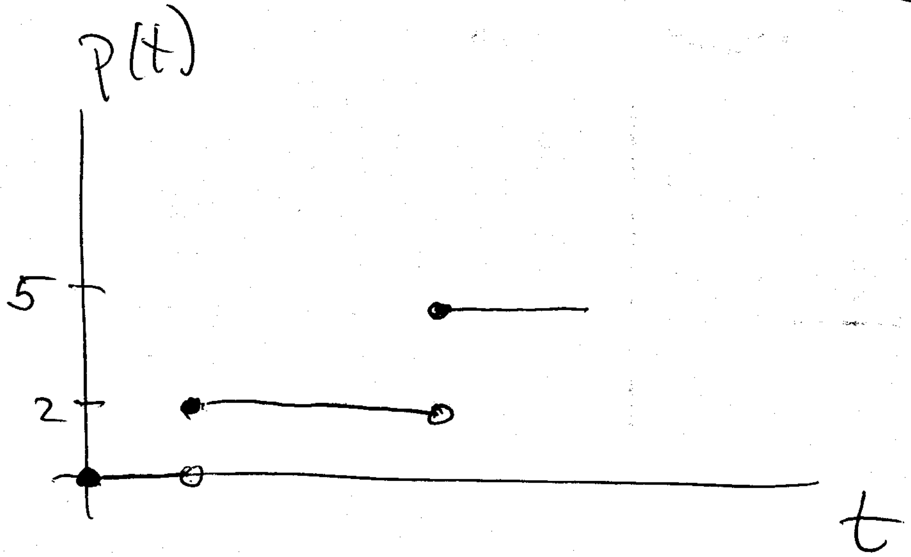
1b)



c)



d)



e.g. $f(x) = \dfrac{x^2-9x+14}{x^2-5x+6}$

vertical asymptotes:

$x^2-5x+6 = 0$

$(x-2)(x-3) = 0$

$x=2 \quad x=3$ plug into numerator

$(2)^2-9(2)+14 = 4-18+14 = 0$ ($x=2$ is not necessarily a v. asymp.)

$(3)^2-9(3)+14 = 9-27+14 = 6$ $\boxed{x=3 \text{ is a vert. asymp.}}$

$x=2$: Find $\lim\limits_{x \to 2} \dfrac{x^2-9x+14}{x^2-5x+6}$

$= \lim\limits_{x\to 2} \dfrac{(x-7)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} = \lim\limits_{x \to 2} \dfrac{x-7}{x-3} = 5$

$\boxed{x=2 \text{ is } \underline{\text{not}} \text{ a vert. asymp}}$

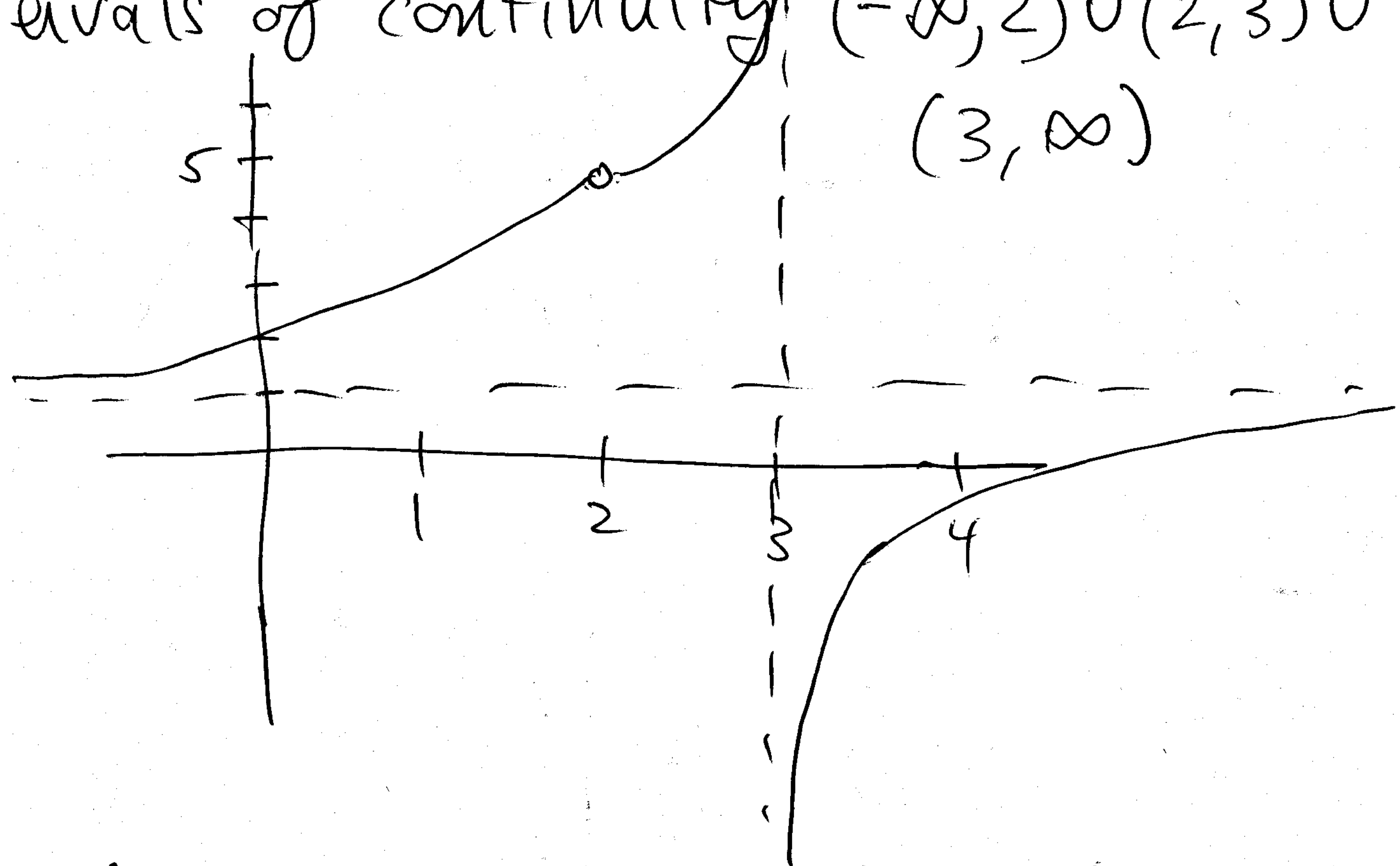
Where is $f(x)$ continuous/discontinuous?

Points of discontinuity: $x=3$ ($f(3)$ does not exist.)

$x=2$ ($f(2)$ does not exist)

$f(x)$ is continuous at all other points.

Intervals of continuity: $(-\infty,2)\cup(2,3)\cup(3,\infty)$

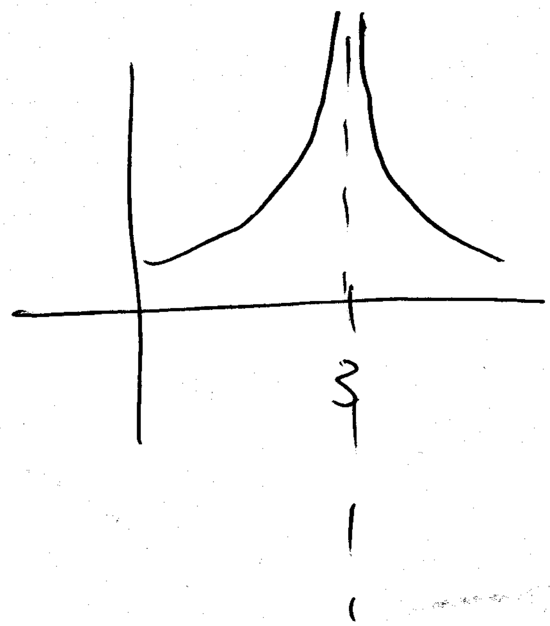


Could I make it continuous at $x=2$?

$$f(x)=\begin{cases} \dfrac{x^2-9x+14}{x^2-5x+6} & \text{if } x\neq 2 \\ 5 & \text{if } x=2 \end{cases}$$ is continuous at $x=2$.

Could I make it continuous at $x=3$?

No. Impossible.

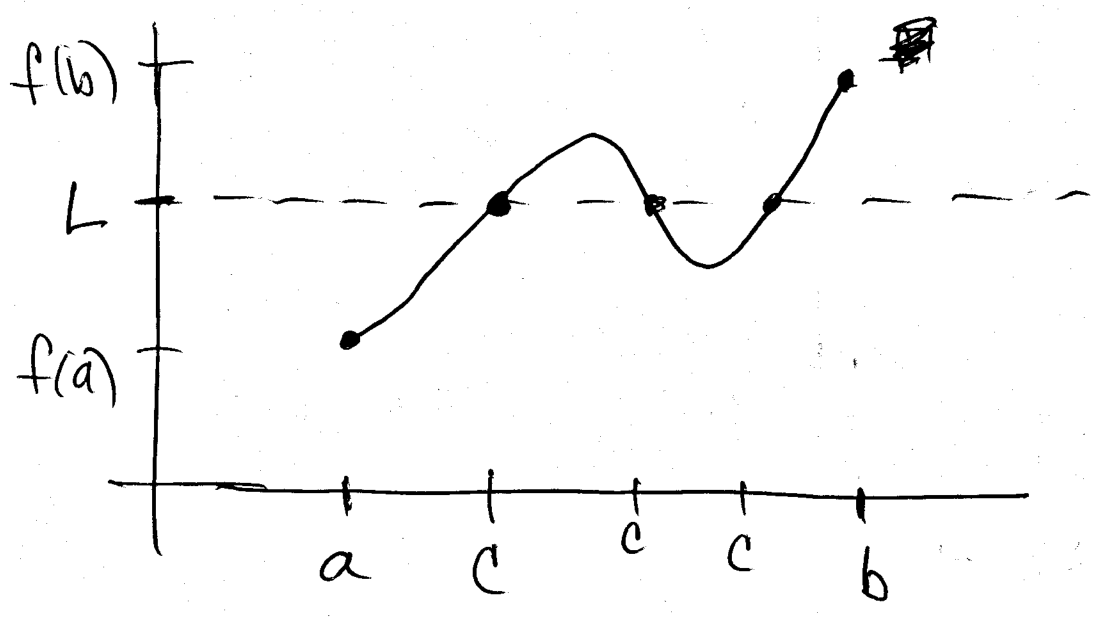


$$\lim_{x\to 3} f(x) = \infty$$

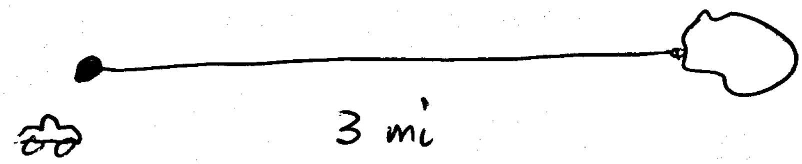
limit does not exist.

discont is not removable

Intermediate Value Thm.



80)



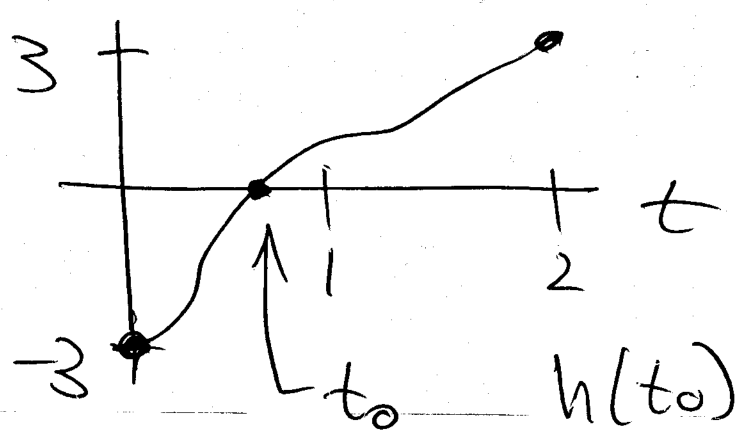
$f(t) =$ dist from car t hrs after start on Fri.

$g(t) =$ " " " " " Sun.

$f(0) = 0 \quad f(2) = 3 \quad g(0) = 3 \quad g(2) = 0$

$h(t) = f(t) - g(t)$

$h(0) = -3 \qquad h(2) = 3$



$h(t_0) = 0$ at some time t_0.