Quiz 2: 2.3, 2.4

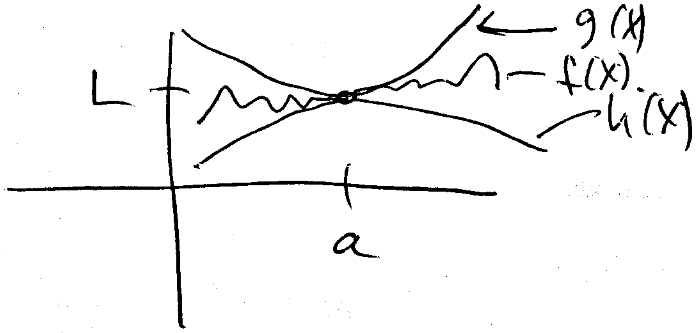
$$\lim_{x \to a} f(x) = L$$

① $\lim_{x \to a} f(x) = f(a)$ for (for example) polynomials, rational functions (when denom $\neq 0$), rational powers of rational functions, also trig functions, exponential functions and log functions (where they are defined).

② Algebraic simplification
(e.g. If p, q are polynomials and $p(a)=q(a)=0$ then p and q have a common factor of $(x-a)$, so

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \lim_{x \to a} \frac{\cancel{(x-a)}\, h(x)}{\cancel{(x-a)}\, r(x)} = \lim_{x \to a} \frac{h(x)}{r(x)}.$$)

③ Sandwich Theorem.



$h(x) \le f(x) \le g(x)$

$\lim_{x \to a} h(x) = \lim_{x \to a} g(x) = L$

then $\lim_{x \to a} f(x) = L$

e.g. $\lim_{x\to 0} \frac{\sin(x)}{x}$

We showed: $\cos(x) \le \frac{\sin(x)}{x} \le \frac{1}{\cos(x)}$

$\lim_{x\to 0} \cos(x) = \cos(0) = 1$

$\lim_{x\to 0} \frac{1}{\cos(x)} = \frac{1}{\cos(0)} = 1$

$\therefore \lim_{x\to 0} \frac{\sin(x)}{x} = 1$

④ Numerically (allows you to guess the limit)

e.g. $\lim_{x\to 0} \frac{2^x - 1}{x}$

x	.1	.05	.01	.005	.001
$\frac{2^x-1}{x}$	.	.	.	.	.

(fill this in)

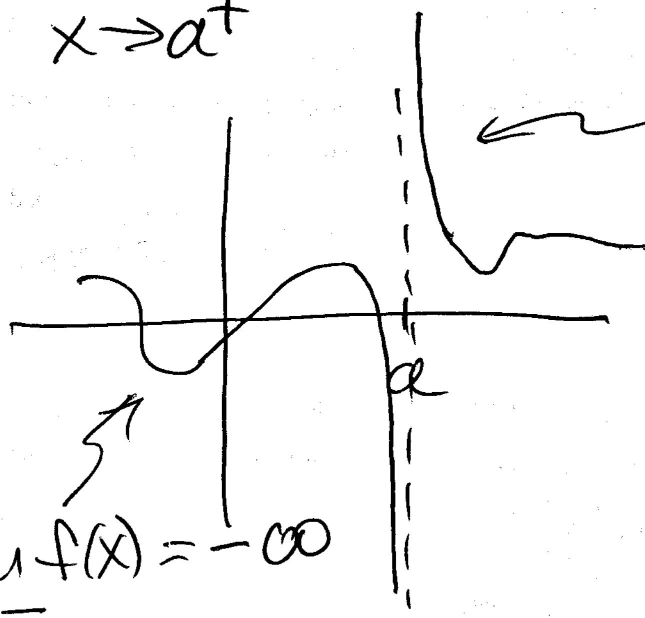
Also talked about one-sided limits

$\lim_{x\to a^+} f(x) = L \qquad \lim_{x\to a^-} f(x) = M.$

2.4 Infinite Limits

$$\lim_{x \to a} f(x) = \pm\infty$$

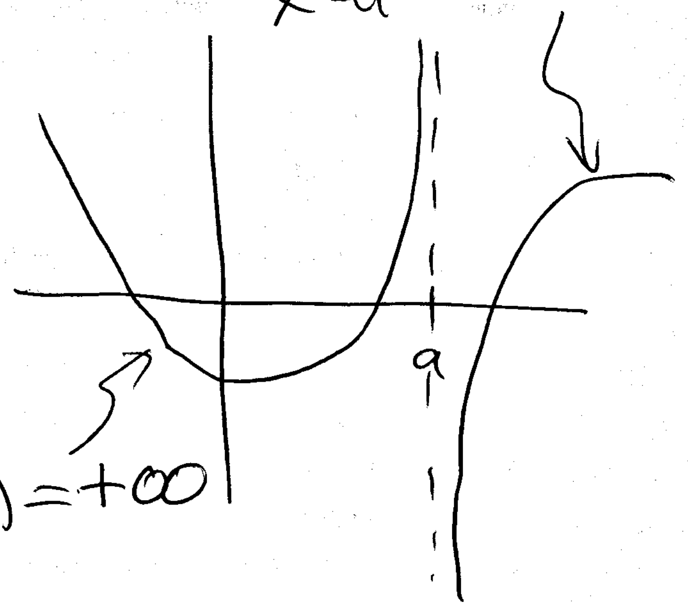
or $\lim_{x \to a^+} f(x) = \pm\infty$, $\lim_{x \to a^-} f(x) = \pm\infty$



$\lim_{x \to a^+} f(x) = +\infty$

$\lim_{x \to a^-} f(x) = -\infty$

$\lim_{x \to a^+} f(x) = -\infty$

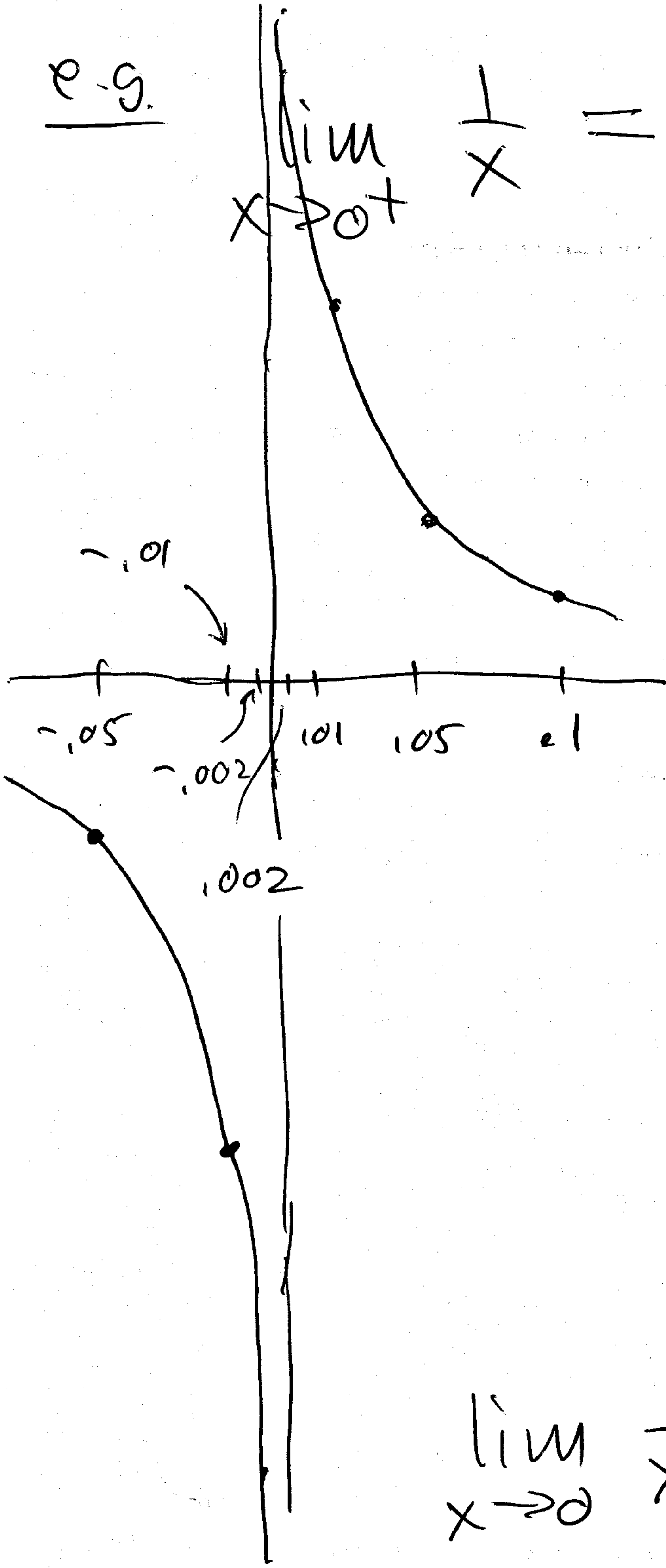


$\lim_{x \to a^-} f(x) = +\infty$

We say $x = a$ is a <u>vertical asymptote</u> of $f(x)$.

e.g. $\lim_{x \to 0^+} \frac{1}{x} = \infty$

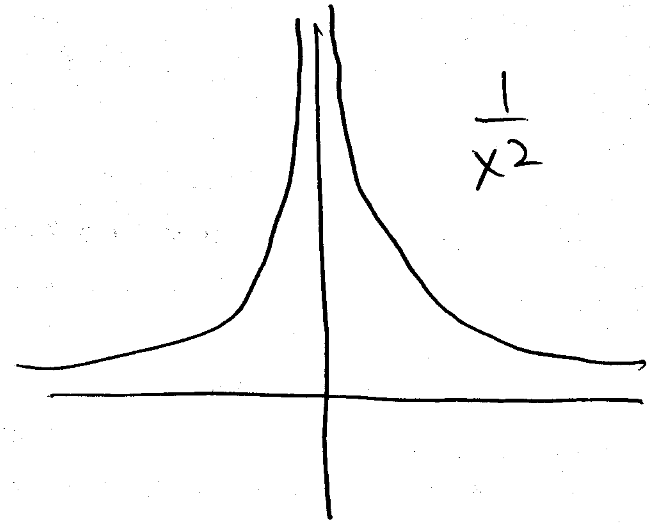
x	$\frac{1}{x}$
.1	10
.05	20
.01	100
.002	500
.0001	10000



$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

x	$1/x$
-.1	-10
-.05	-20
-.002	-500
-.0001	-10000

$\lim_{x \to 0} \frac{1}{x}$ does not exist eg $\lim_{x \to 0^+} \frac{1}{x^2} = \infty$



$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$

We say $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

Fact: For rational functions $f(x) = \frac{P(x)}{Q(x)}$

If $P(a) \neq 0$ and $Q(a) = 0$ then $f(x)$ has a vertical asymptote at $x = a$.

eg #8) $f(x) = \frac{x}{(x^2-2x-3)^2} = \frac{x}{(x-3)^2(x+1)^2}$

Find v. asymptotes: $x=3$ $x=-1$

$\lim_{x \to 3^-} f(x) = \infty$

If $x < 3$ but close to 3 is $f(x)$ pos or neg?

$\frac{+}{(+)(+)} = (+)$

$$\lim_{x \to -1^+} f(x) = -\infty$$

If $x > -1$
but close to
-1, is $f(x)$ pos
or neg?

$$\frac{(-)}{(+)(+)} = (-)$$

$$\lim_{x \to 0} \frac{x}{(x-3)^2(x+1)^2} = \frac{0}{(-3)^2(1)^2} = 0$$

#20) $\lim_{t \to 5} \frac{4t^2-100}{t-5} = \lim_{t \to 5} 4(t+5) = 40$.

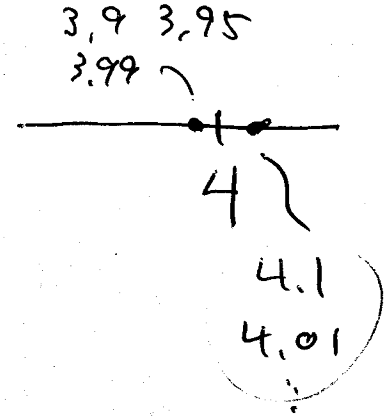
$$\frac{4t^2-100}{t-5} = \frac{4(t^2-25)}{t-5} = \frac{4(t+5)\cancel{(t-5)}}{\cancel{t-5}}$$

$$= 4(t+5) \text{ if } t \neq 5$$

#22) $\lim_{z \to 4} \frac{z-5}{(z^2-10z+24)^2}$

$$= \lim_{z \to 4} \frac{z-5}{(z-4)^2(z-6)^2} = -\infty.$$

$$\lim_{z \to 4^+} \frac{z-5}{(z-4)^2(z-6)^2} = -\infty$$



$$\lim_{z \to 4^-} \frac{z-5}{(z-4)^2(z-6)^2} = -\infty$$

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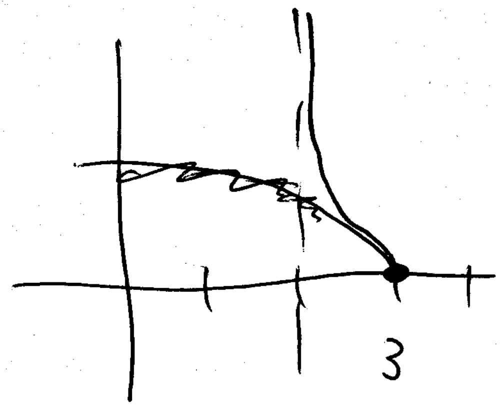
$$\lim_{x \to 3^-} \sqrt{\frac{x-3}{2-x}} = 0$$



$$\lim_{x \to 3^+} \sqrt{\frac{x-3}{2-x}}$$ does not exist.

if $x<3$ but close to 3 $\quad \frac{(-)}{(-)} = (+)$

is $\frac{x-3}{2-x}$ pos or neg?

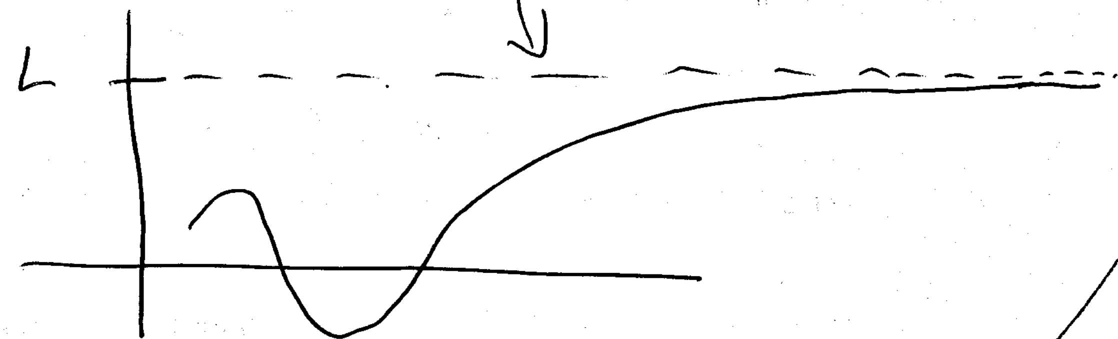
if $x>3$ but close to 3

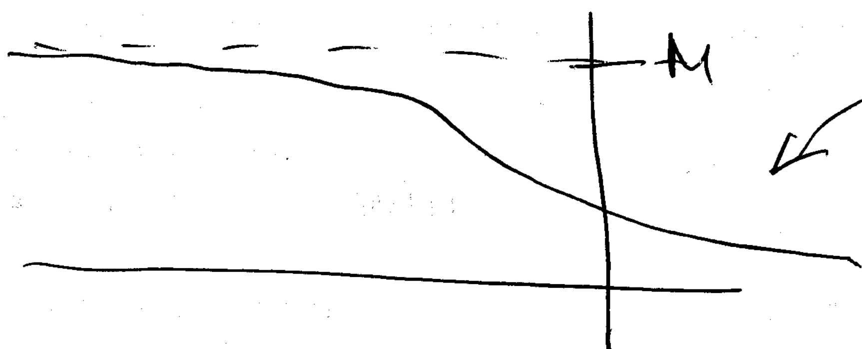
$\frac{x-3}{2-x}$ is $\frac{(+)}{(-)} = (-)$.

2.5 Limits at infinity

$$\lim_{x \to \infty} f(x) = L \qquad \lim_{x \to -\infty} f(x) = M$$

Graphically

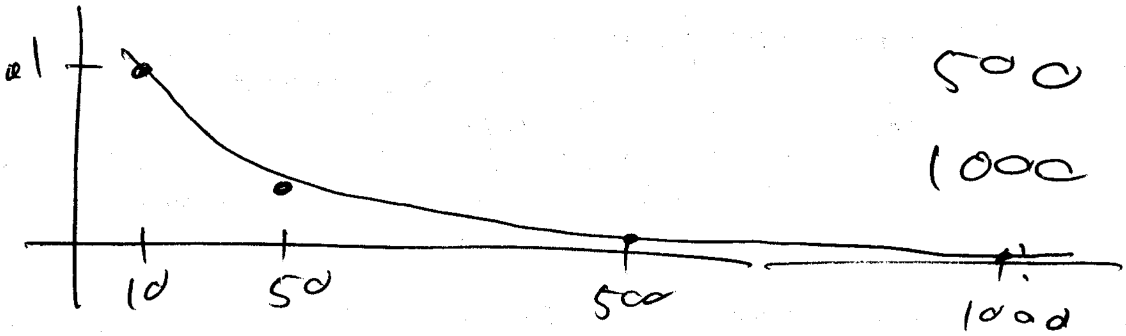




In these cases, $y = L$ and $y = M$ are horizontal asymptotes of $f(x)$.

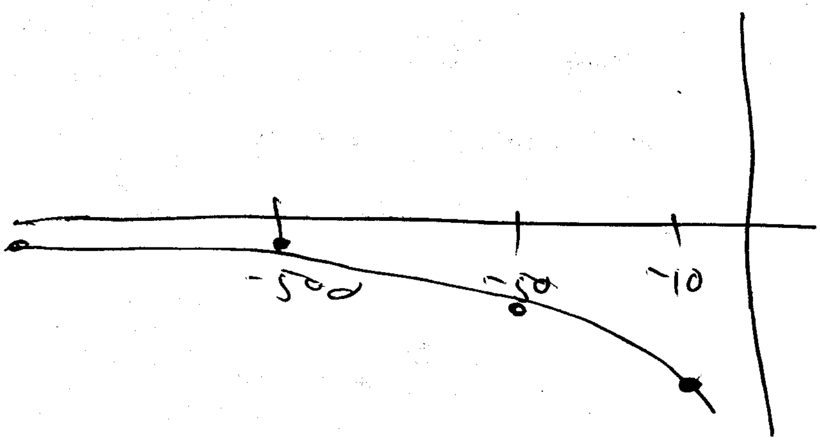
e.g. $\lim_{x \to \infty} \frac{1}{x} = 0$

x	$\frac{1}{x}$
10	.1
50	.02
500	.002
1000	.001



$\lim_{x \to -\infty} \frac{1}{x} = 0$

x	$\frac{1}{x}$
-10	$-.1$
-500	$-.002$
-100000	$-.00001$
-200000	$-.000005$
$\vdots$	



#10 $\lim_{x \to \infty} \left(5 + \frac{1}{x} + \frac{10}{x^2}\right) =$

$\lim_{x \to \infty} 5 + \lim_{x \to \infty} \frac{1}{x} + \lim_{x \to \infty} \frac{10}{x^2} = 5 + 0 + 0$

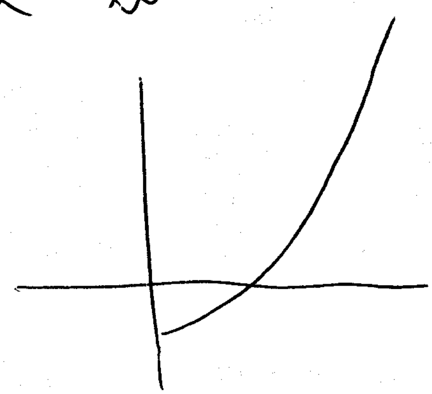
$= 5$

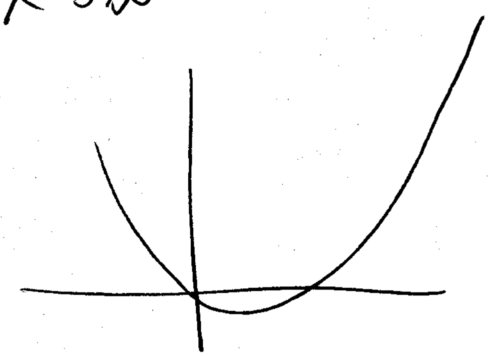
e.g. $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

x	$\frac{\sin x}{x}$
.1	
.01	
.001	
.0005	
⋮	

e.g. $\lim_{x \to \infty} \frac{4x^2-7}{8x^2+5x+2} = \lim_{x \to \infty} \frac{4-\frac{7}{x^2}^{\to 0}}{8+\cancel{\frac{5}{x}}^{\to 0}+\cancel{\frac{2}{x^2}}^{\to 0}} = \frac{4}{8} = \frac{1}{2}$

$\lim_{x \to \infty} 4x^2-7 = \infty$ $\qquad$ $\lim_{x \to \infty} 8x^2+5x+2 = \infty$





$$\frac{4x^2-7}{8x^2+5x+2} \cdot \left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) = \frac{4-\frac{7}{x^2}}{8+\frac{5}{x}+\frac{2}{x^2}}$$

e.g. $\lim_{x\to\infty} \dfrac{-x^3+1}{2x+8} = \lim_{x\to\infty} \dfrac{-1+\frac{1}{x^3}}{\frac{2}{x^2}+\frac{8}{x^3}} = -\infty$

$$\frac{-x^3+1}{2x+8}\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right) = \frac{-1+\frac{1}{x^3}}{\frac{2}{x^2}+\frac{8}{x^3}}$$

eg $\lim_{x\to\infty} \dfrac{4x^2}{2x^3+\sqrt{9x^6+15x^4}} =$

$$\lim_{x\to\infty} \frac{4x^2}{2x^3+\sqrt{9x^6}} = \lim_{x\to\infty} \frac{4x^2}{2x^3+3x^3}$$

$$= \lim_{x\to\infty} \frac{4\cancel{x^2}}{5x^{\cancel{3}}} = \lim_{x\to\infty} \frac{4}{5x} = 0$$