Quiz 1 - Tuesday Sec 2.1, 2.2

$$\lim_{x \to a} f(x) = L \qquad \lim_{t \to b} h(t) = M$$

① limits give meaning to expressions that evaluate to $0/0$.

② $\lim_{x \to a} f(x)$ depends on values of $f(x)$ for $x$ <u>near</u> a but <u>not equal to</u> a

③ limits do not have to exist.

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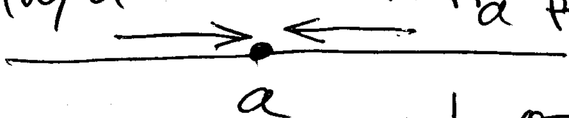
One-sided limits

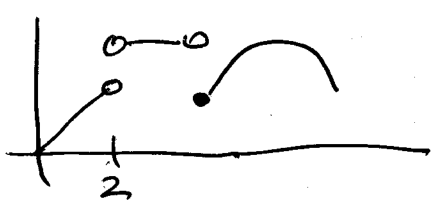
$$\lim_{x \to a^+} f(x) \qquad \lim_{x \to a^-} f(x)$$

approaching a from left

approaching a from right



<u>eg</u> #18

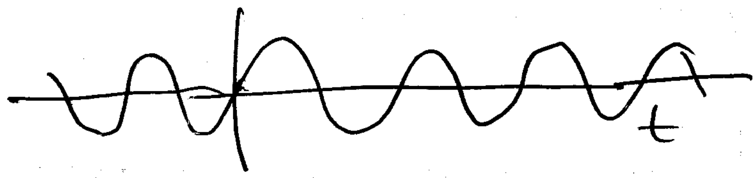


<u>Fact</u>: $\lim_{x \to a} f(x)$ exists if and only if $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ both exist and are equal.

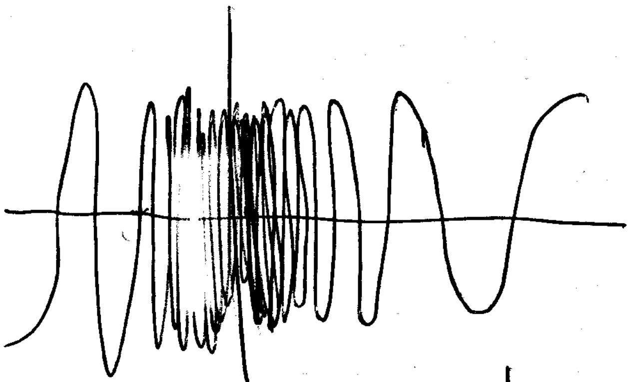
How else can a limit fail to exist?

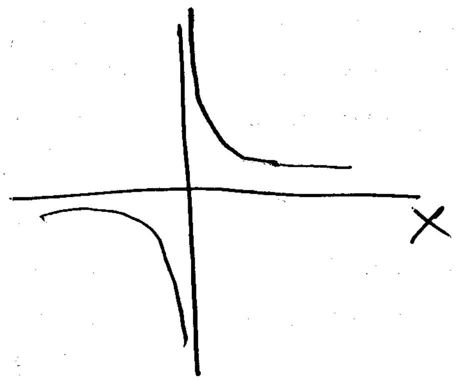
① oscillation

$\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist



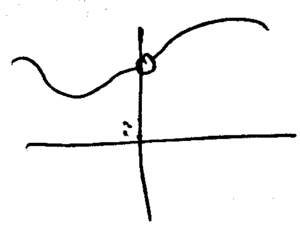

$\sin(t)$





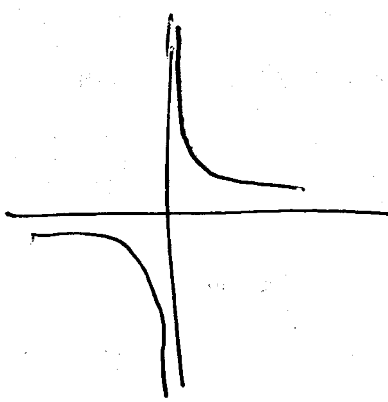

$\frac{1}{x}$

In fact even $\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right)$ does not exist. ($\lim_{x\to 0^-} \sin\left(\frac{1}{x}\right)$ also)

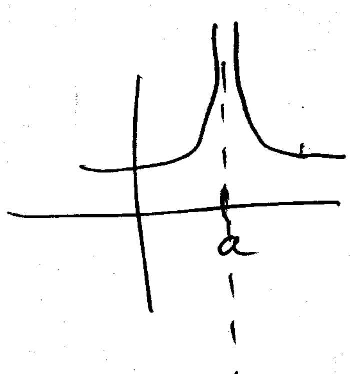


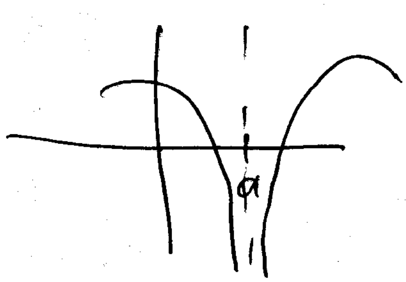
② infinite limits

$\lim_{x\to 0} \frac{1}{x}$ does not exist



vertical asymptote



## 2.3 Computing limits

1. Sometimes $\lim_{x \to a} f(x) = f(a)$. When?

a. If $f(x)$ is a polynomial, i.e.

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

e.g. $f(x) = 3x - 7$

$f(t) = 4t^6 - 6t^5 + t + 1$ etc.

$$\lim_{x \to 5} (3x - 7) = 3(5) - 7 = 8$$

$$\lim_{x \to 1} (4x^6 - 6x^5 + x + 1) = 4 - 6 + 1 + 1 = 0.$$

b. If $f(x)$ is a rational function, i.e.

$f(x) = \dfrac{P(x)}{Q(x)}$, $P, Q$ are polynomials.

e.g. $f(x) = \dfrac{2x + 4}{6x^2 + 2x + 1}$

$$f(x) = \frac{3x^2 - 4x}{5x^3 - 36}$$

$$\lim_{x\to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}$$ as long as $Q(a) \neq 0$.

e.g. $$\lim_{t\to 2} \frac{3t^2-7t}{2t-3} = \frac{3(4)-7(2)}{2(2)-3} = \frac{-2}{1} = -2$$

c. If $f(x)$ is a rational power of a polynomial or rational function

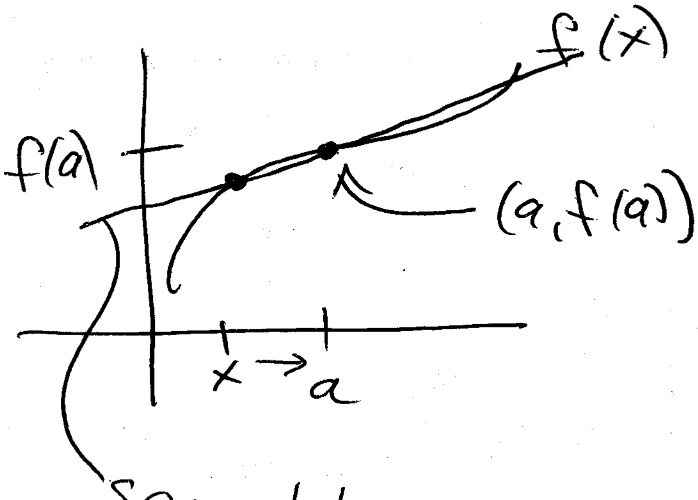
e.g. $$\lim_{x\to 3} \sqrt[3]{x^2-10} = \lim_{x\to 3} (x^2-10)^{1/3}$$
$$= (3^2-10)^{1/3} = (-1)^{1/3} = -1.$$

eg #30) 2.3 $$\lim_{h\to 0} \frac{3}{\sqrt{16+3h}+4} = \frac{3}{\sqrt{16}+4}$$
$$= \frac{3}{4+4} = \frac{3}{8}$$

Review

2.1 #4

slope of tangent line ~~&~~ to $f(x)$ at $(a, f(a))$




secant line slope $= \dfrac{f(x)-f(a)}{x-a}$

slope of tangent line

$$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

2. Algebraic simplification

#38) $\displaystyle\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$ (evaluates to $\frac{0}{0}$)

<u>Fact</u>: Suppose $\frac{P(x)}{Q(x)}$ and that $P(a)=0$ and $Q(a)=0$, then P and Q have a common factor of $(x-a)$.

$$x^2-2x-3 = (x-3)(x+1)$$

so $$\frac{x^2-2x-3}{x-3} = \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = x+1$$ $\boxed{\text{IF } x \neq 3}$

$$\lim_{x\to 3}\frac{x^2-2x-3}{x-3} = \lim_{x\to 3}\frac{(x-3)(x+1)}{x-3}$$

$$= \lim_{x\to 3}(x+1) = 4$$

#44) $\lim\limits_{h\to 0} \dfrac{\left(\frac{1}{5+h} - \frac{1}{5}\right)}{h}$

$$\frac{\frac{1}{5+h}\cdot 5 - \frac{1}{5}}{h} = \frac{\frac{5-(5+h)}{5(5+h)}}{h} = \frac{\frac{-h}{5(5+h)}}{h}$$

$$= \frac{-h}{5(5+h)} \cdot \frac{1}{h} = \frac{-\cancel{h}}{5\cancel{h}(5+h)} = \frac{-1}{5(5+h)} \quad \boxed{\text{IF } h \neq 0}$$

$$\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{-1}{5(5+h)} = \frac{-1}{25} \;//$$

# 47) $\lim\limits_{h\to 0} \dfrac{\sqrt{16+h}-4}{h} = \lim\limits_{h\to 0} \dfrac{1}{\sqrt{16+h}+4} = \dfrac{1}{\sqrt{16}+4}$

$$= 1/8 \;//$$

$$\frac{\sqrt{16+h}-4}{h} \cdot \frac{\sqrt{16+h}+4}{\sqrt{16+h}+4} = \frac{(\sqrt{16+h})^2 - 4^2}{h(\sqrt{16+h}+4)}$$

$$= \frac{16+h-16}{h(\sqrt{16+h}+4)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{16+h}+4)} = \frac{1}{\sqrt{16+h}+4}$$

$\boxed{\text{IF } h \neq 0}$

#16) $\displaystyle\lim_{t\to a}\frac{\sqrt{3t+1}-\sqrt{3a+1}}{t-a}=\frac{3}{2\sqrt{3a+1}}$.

$$\frac{\sqrt{3t+1}-\sqrt{3a+1}}{t-a}\cdot\frac{\sqrt{3t+1}+\sqrt{3a+1}}{\sqrt{3t+1}+\sqrt{3a+1}}$$

$$=\frac{(\sqrt{3t+1})^2-(\sqrt{3a+1})^2}{(t-a)(\sqrt{3t+1}+\sqrt{3a+1})}=\frac{3t+1-(3a+1)}{(t-a)(\sqrt{3t+1}+\sqrt{3a+1})}$$

$$=\frac{3\cancel{(t-a)}}{\cancel{(t-a)}(\sqrt{3t+1}+\sqrt{3a+1})}=\frac{3}{(\sqrt{3t+1}+\sqrt{3a+1})}$$

$\boxed{\text{if } t\neq a}$

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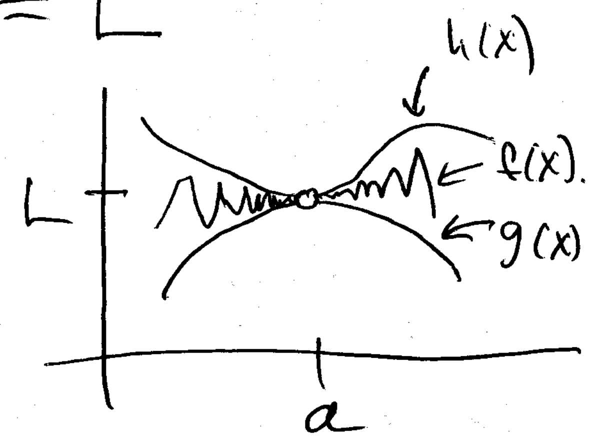
3. Sandwich Theorem

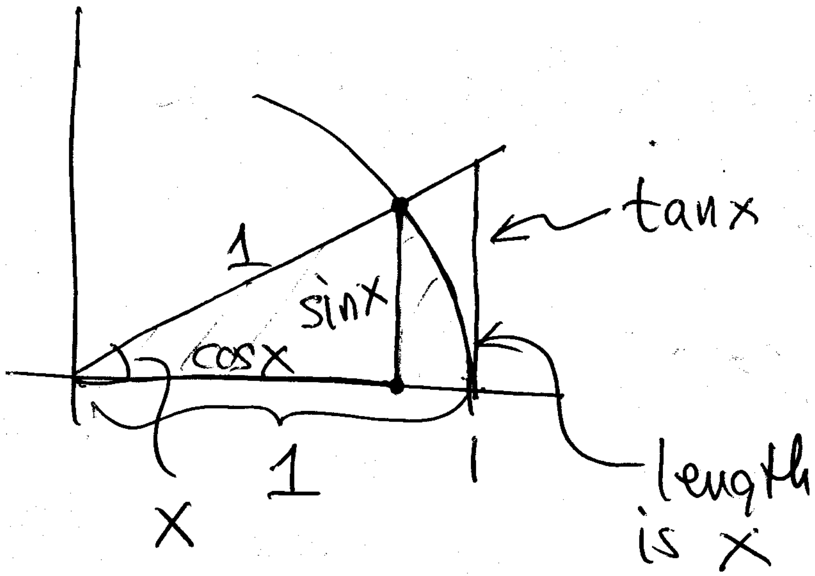
If $g(x)\leq f(x)\leq h(x)$ and

$\displaystyle\lim_{x\to a}g(x)=\lim_{x\to a}h(x)=L$

then $\displaystyle\lim_{x\to a}f(x)=L$.



e.g. $\boxed{\lim_{x \to 0} \frac{\sin(x)}{x} = 1}$ $\qquad \frac{0}{0}$




$$\begin{matrix}\text{area of} \\ \text{small } \Delta\end{matrix} \leq \begin{matrix}\text{area of} \\ \text{sector}\end{matrix} \leq \begin{matrix}\text{area of} \\ \text{big } \Delta\end{matrix}$$

$$\frac{1}{2}\,\frac{\cos x}{\sin x} \leq (\cancel{\pi})\left(\frac{x}{2\cancel{\pi}}\right) \leq \frac{1}{2}\,\frac{\sin x}{\cos x}$$

$$\cos x \leq \frac{\sin x}{x} \leq \frac{1}{\cos x}$$

we know $\lim_{x \to 0} \cos x = \lim_{x \to 0} \frac{1}{\cos x} = 1$