MATH 111 – EXAM 4 – SOLUTIONS

1. (a) $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 40 \\ 33 \\ 30 \\ 26 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 14 \\ 14 & 54 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 129 \\ 429 \end{bmatrix}$$ //

(b) $$\begin{bmatrix} 4 & 14 \\ 14 & 54 \end{bmatrix}^{-1} = \frac{1}{20} \begin{bmatrix} 54 & -14 \\ -14 & 4 \end{bmatrix}$$

$D = (4)(54) - (14)(14) = 20$

$$= \begin{bmatrix} 2.7 & -.7 \\ -.7 & .2 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 2.7 & -.7 \\ -.7 & .2 \end{bmatrix} \begin{bmatrix} 129 \\ 429 \end{bmatrix} \approx \begin{bmatrix} 48 \\ -4.5 \end{bmatrix}$$

$y = -4.5x + 48$ //

(c) $y = -4.5(6.5) + 48$

$= 18.75$ or 18,750 fish

(d) $35 = -4.5x + 48$

$-4.5x = -13$

$x \cong 2.9$ tons of pollutant

2 (a)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$-1\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & -2 & 0 & -1 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & 1 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \quad y = \frac{1}{2}x + \frac{1}{2}x^2 //$$

(b)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -2 & 4 & -8 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

(c)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ 1 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ 1 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 6 \\ -2 & 5 & -9 \\ 6 & -9 & 18 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} //$$

$$y = a_0 + a_1 x + a_2 x^2$$

3. $\begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1/2 \end{bmatrix}$ $R = \begin{bmatrix} 1/2 \end{bmatrix}$ $S = \begin{bmatrix} 1/3 \\ 1/6 \end{bmatrix}$

$$(I-R) = [1 - 1/2] = [1/2]$$

$$(I-R)^{-1} = [2]$$

$$S(I-R)^{-1} = \begin{bmatrix} 1/3 \\ 1/6 \end{bmatrix} [2] = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \end{bmatrix} = \text{stable matrix}$$

//

4. (a) R and F are absorbing states.

(b) $\frac{7}{8}$ of Top Managers eventually retire.
$\frac{1}{8}$ of Top Managers eventually are fired.
$\frac{4}{5}$ of Middle Managers eventually retire
$\frac{1}{5}$ of Middle Managers eventually are fired.