MATH 111 – EXAM 4 – 13 JUNE 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. An experimental study of fish populations in large lakes in which a particular pollutant is dumped produced the following data. Here x is the number of tons of the pollutant introduced into the water and y is the population of fish in thousands within one half mile of the source of the pollutant.

x	y
2	40
3	33
4	30
5	26

(a) (10 pts.) Set up the normal equations giving the least squares line best fitting the above data.

(b) (10 pts.) Solve the equations you found in part (a) and write down the best fit line for these data.

(c) (5 pts.) Use the line you found in part (b) to estimate the fish population if 6.5 tons of the pollutant are dumped into the lake.

(d) (5 pts.) Suppose that there are 35000 fish within a half mile of the source of the pollutant. Use the line you found in part (b) to estimate the number of tons of the pollutant that have been introduced into the lake.

2. (10 pts. each)

(a) Find the polynomial that interpolates the points $(1, 1)$, $(0, 0)$, and $(-1, 0)$. Be sure to show all work. You may use your calculator only to do basic arithmetic.

(b) Set up the system of equations (in matrix form where the matrices are given explicitly) giving the polynomial that interpolates the points $(1, 1)$, $(0, 0)$, $(-1, 0)$, and $(-2, 1)$. (Note that this is the same set of points as in part (a) but with the point $(-2, 1)$ added.) DO NOT SOLVE.

(c) Set up the system of normal equations (in matrix form where the matrices are given explicitly) giving the least squares polynomial of degree 2 that best fits the points $(1, 1)$, $(0, 0)$, $(-1, 0)$, and $(-2, 1)$. DO NOT SOLVE.

3. (10 pts.) Find the stable matrix for the absorbing stochastic matrix $\begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1/2 \end{bmatrix}$.

4. (5 pts. each) In a certain company, managers are classified as top managers (T) or middle managers (M). Each year the managers are retired (R), fired (F), promoted, demoted, or remain at their level. This situation is described by an absorbing stochastic matrix, whose stable matrix is given below.

	R	F	T	M
R	1	0	7/8	4/5
F	0	1	1/8	1/5
T	0	0	0	0
M	0	0	0	0

(a) Identify the absorbing states in this system.

(b) Give an interpretation of the numbers in the 2×2 matrix in the upper right corner of the given matrix.