MATH 111 - EXAM 3 - SOLUTIONS

1. (a)

$$\begin{array}{c} \\ M \\ Ag \end{array}\begin{array}{c} \begin{array}{cc} M & Ag \end{array} \\ \begin{bmatrix} .30 & .20 \\ .10 & .20 \end{bmatrix} \end{array}$$

(b) $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .30 & .20 \\ .10 & .20 \end{bmatrix}$

$= \begin{bmatrix} .70 & -.20 \\ -.10 & .80 \end{bmatrix}$

$(I-A)^{-1} = \frac{1}{.54}\begin{bmatrix} .80 & .20 \\ .10 & .70 \end{bmatrix} \approx \begin{bmatrix} 1.48 & .37 \\ .19 & 1.30 \end{bmatrix}$

$D = (.70)(.80) - (-.10)(-.20)$

$= .54$

(c) $D = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$ $(I-A)^{-1}D = \begin{bmatrix} 1.48 & .37 \\ .19 & 1.30 \end{bmatrix}\begin{bmatrix} 100 \\ 50 \end{bmatrix}$

$= \begin{bmatrix} 166.5 \\ 84 \end{bmatrix}$ Output is \$166,500 from M and \$184,000 from Ag. //

2. (a)

$$\begin{array}{c} \\ S \\ W \end{array}\begin{array}{c} \begin{array}{cc} S & W \end{array} \\ \begin{bmatrix} .4 & .3 \\ .6 & .7 \end{bmatrix} \end{array} = A.$$

(b) $X_0 = \begin{bmatrix} .1 \\ .9 \end{bmatrix}_0$ $X_1 = AX_0 = \begin{bmatrix} .4 & .3 \\ .6 & .7 \end{bmatrix}\begin{bmatrix} .1 \\ .9 \end{bmatrix}$

$= \begin{bmatrix} .31 \\ .69 \end{bmatrix}_1$ //

$X_2 = AX_1 = \begin{bmatrix} .4 & .3 \\ .6 & .7 \end{bmatrix}\begin{bmatrix} .31 \\ .69 \end{bmatrix}$

$= \begin{bmatrix} .331 \\ .669 \end{bmatrix}_2$ //

∴ After 1 week:
3,100,000 sick
6,900,000 well //

After 2 weeks:
3,310,000 sick
6,690,000 well. //

(c) $A - I = \begin{bmatrix} .4 & .3 \\ .6 & .7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -.6 & .3 \\ .6 & -.3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ -.6 & .3 & 0 \\ .6 & -.3 & 0 \end{bmatrix} \xrightarrow{-6} \begin{bmatrix} 1 & 1 & 1 \\ -6 & 3 & 0 \\ 6 & -3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 9 & 6 \\ 0 & -9 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{2}{3} \\ 0 & -9 & -6 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore X = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} //$$

In the long run, $\frac{1}{3}$ of the population will have a cold, $\frac{2}{3}$ will be well. //

3 (a) $\begin{bmatrix} .1 & .5 & .6 \\ 0 & .3 & .4 \\ .9 & .2 & 0 \end{bmatrix} \begin{bmatrix} .1 & .5 & .6 \\ 0 & .3 & .4 \\ .9 & .2 & 0 \end{bmatrix} = \begin{bmatrix} .55 & .32 & .26 \\ .36 & .17 & .12 \\ .09 & .51 & .62 \end{bmatrix}$

Since A^2 has positive entries, A is regular.

(b) $\begin{bmatrix} 1 & .8 \\ 0 & .2 \end{bmatrix} \begin{bmatrix} 1 & .8 \\ 0 & .2 \end{bmatrix} = \begin{bmatrix} 1 & .96 \\ 0 & .04 \end{bmatrix}$

$\begin{bmatrix} 1 & .96 \\ 0 & .04 \end{bmatrix} \begin{bmatrix} 1 & .8 \\ 0 & .2 \end{bmatrix} = \begin{bmatrix} 1 & .992 \\ 0 & .008 \end{bmatrix}$

No matter how large n is, the first column of A^n is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ so matrix is <u>not</u> regular.