MATH 111 – EXAM 3 – 6 JUNE 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (10 pts. each) A simple economy has two sectors, manufacturing and agriculture. Suppose that each \$1 of output from the manufacturing sector requires \$.30 worth of input from the manufacturing sector and \$.10 from the agriculture sector, and that each \$1 of output from the agriculture sector requires \$.20 worth of input from the manufacturing sector and \$.20 from the agriculture sector.

(a) Write down the input–output matrix A for this economy.

(b) Determine the matrix $(I - A)^{-1}$ with all entries correctly rounded to two decimal places.

(c) At what level of output should each sector produce to meet a demand of \$50,000 worth of agricultural output and \$100,000 worth of manufacturing output?

2. (10 pts. each) Suppose that people in a certain city are catching cold. It is observed that after one week 40% of the people who were sick are still sick. Of the people who were well, 30% are sick after one week.

(a) Find a transition matrix, A, that describes this situation.

(b) If out of a population of 10 million people, 10% are sick, how many will be sick after one week? After two weeks?

(c) Find the stable distribution of the stochastic matrix A that you found in part (a). Give an interpretation of these numbers.

3. (10 pts. each) Determine whether each of the following matrices is a *regular* stochastic matrix. Explain your answers. (Hint: Remember that a stochastic matrix is regular if some power of it has all positive entries.)

(a) $\begin{bmatrix} .1 & .5 & .6 \\ 0 & .3 & .4 \\ .9 & .2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & .8 \\ 0 & .2 \end{bmatrix}$