

Exam 4 - 8.3, 1.5, Ch 2-6 (suppl notes.) ⁶⁻¹²⁻²⁰¹³

Interpolation vs. Least squares.

Example 4.2 p 30 (notes)

X	1.82	3.69	4.88	2.29	5.23
y	2.03	4.56	3.73	3.28	2.94

↑ ↑ ↑

x = time (sec)

y = height (m)

$$y = a_0 + a_1 x + a_2 x^2$$

$$\begin{bmatrix} 1 & 1.82 & (1.82)^2 \\ 1 & 4.88 & (4.88)^2 \\ 1 & 2.29 & (2.29)^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.03 \\ 3.73 \\ 3.28 \end{bmatrix}$$

$A \quad \quad \quad \bar{X} = b$

$$\bar{X} = A^{-1} b$$

Take all 5 points.

① Do a degree 4 interpolant.

② Do a least squares quadratic

$$\begin{bmatrix} 1 & 1.82 & (1.82)^2 \\ 1 & 3.69 & (3.69)^2 \\ 1 & 4.88 & (4.88)^2 \\ 1 & 2.29 & (2.29)^2 \\ 1 & 5.23 & (5.23)^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.03 \\ 4.56 \\ 3.73 \\ 3.28 \\ 2.94 \end{bmatrix}$$

$$A \quad X = b$$

Normal equations: $A^T A X = A^T b$.

" 3 eqns, 3 unknowns.

My Math Lab Quiz 4

(1)

$$\begin{array}{c} A \quad B \quad C \quad D \\ A \left[\begin{array}{cccc} .7 & .5 & 0 & 0 \\ .3 & .5 & 0 & .6 \\ 0 & 0 & 1 & .2 \\ 0 & 0 & 0 & .2 \end{array} \right] \end{array}$$

(2)

$$\begin{array}{c} A \quad B \quad C \quad D \\ A \left[\begin{array}{cccc} .6 & 1 & .2 & 0 \\ .1 & 0 & .3 & 0 \\ .1 & 0 & .1 & 0 \\ .2 & 0 & .4 & 1 \end{array} \right] \end{array}$$

$$\begin{array}{c} D \quad A \quad B \quad C \\ A \left[\begin{array}{cccc} 0 & .6 & 1 & .2 \\ 0 & .1 & 0 & .3 \\ 0 & .1 & 0 & .1 \\ 1 & .2 & 0 & .4 \end{array} \right] \end{array}$$

$$\begin{array}{c} D \quad A \quad B \quad C \\ D \left[\begin{array}{cccc} 1 & .2 & 0 & .4 \\ 0 & .6 & 1 & .2 \\ 0 & .1 & 0 & .3 \\ 0 & .1 & 0 & .1 \end{array} \right] \end{array}$$

③

$$\left[\begin{array}{cc|cc} 1 & 0 & .2 & 0 \\ 0 & 1 & .4 & .2 \\ \hline 0 & 0 & .3 & .6 \\ 0 & 0 & .1 & .2 \end{array} \right]$$

$$R = \begin{bmatrix} .3 & .6 \\ .1 & .2 \end{bmatrix}$$

$$S = \begin{bmatrix} .2 & 0 \\ .4 & .2 \end{bmatrix}$$

Fundamental matrix = $(I - R)^{-1}$

$$I - R = \begin{bmatrix} .7 & -.6 \\ -.1 & .8 \end{bmatrix}$$

$$(I - R)^{-1} = \frac{1}{.5} \begin{bmatrix} .8 & +.6 \\ +.1 & .7 \end{bmatrix} = \begin{bmatrix} 1.6 & +1.2 \\ +.2 & 1.4 \end{bmatrix}$$

$$D = (.7)(.8) - (.6)(.1) = .56 - .06 = .5$$

~~$$(I - R)^{-1} S = \begin{bmatrix} 1.6 & 1.2 \\ .2 & 1.4 \end{bmatrix} \begin{bmatrix} .2 & 0 \\ .4 & .2 \end{bmatrix} = \begin{bmatrix} .8 & .24 \\ .6 & .28 \end{bmatrix}$$~~

$$\begin{aligned} & .32 + .48 \\ & .04 + .56 \\ & = \begin{bmatrix} .32 & .24 \\ .68 & .76 \end{bmatrix} \end{aligned}$$

④

$$\begin{matrix} & L & A & P \\ \begin{matrix} L \\ A \\ P \end{matrix} & \begin{bmatrix} 1 & .3 & .1 \\ 0 & .4 & 0 \\ 0 & .3 & .9 \end{bmatrix} \end{matrix}$$

$$R = \begin{bmatrix} .4 & 0 \\ .3 & .9 \end{bmatrix} \quad I - R = \begin{bmatrix} .6 & 0 \\ -.3 & .1 \end{bmatrix}$$

$$(I - R)^{-1} = \frac{1}{.06} \begin{bmatrix} .1 & 0 \\ .3 & .6 \end{bmatrix} = \begin{bmatrix} \frac{10}{6} & 0 \\ \frac{30}{6} & \frac{60}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{3} & 0 \\ 5 & 10 \end{bmatrix} \quad \frac{10}{6} \cdot \frac{3}{10}$$

$$5(I - R)^{-1} = \begin{bmatrix} .3 & .1 \end{bmatrix} \begin{bmatrix} \frac{5}{3} & 0 \\ 5 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

5

⑥

X	1	2	3	4
y	7	6	5	4

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 22 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix}^{-1} = \frac{1}{20} \begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{5} \end{bmatrix}$$

120 - 100

$$\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 22 \\ 50 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

33 - 25

-11 + 10

6

$$y = -x + 8$$