6-10-2013

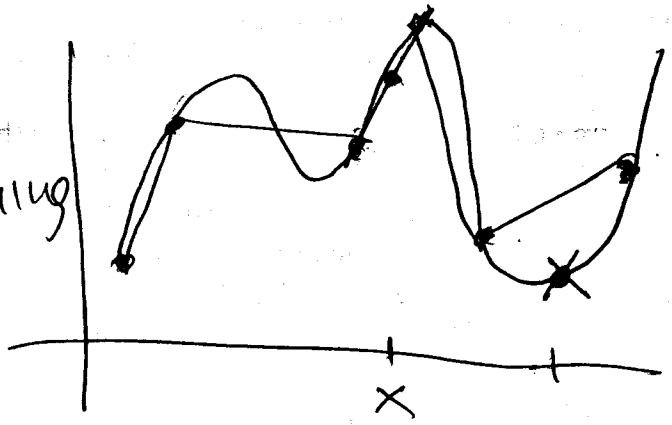
MATLAB Assn 2 due Wednesday
Exam 4 – Thursday

Polynomial Interpolation and Least Squares.

① Interpolation.

Data points:

Q: What is happening in between data points?



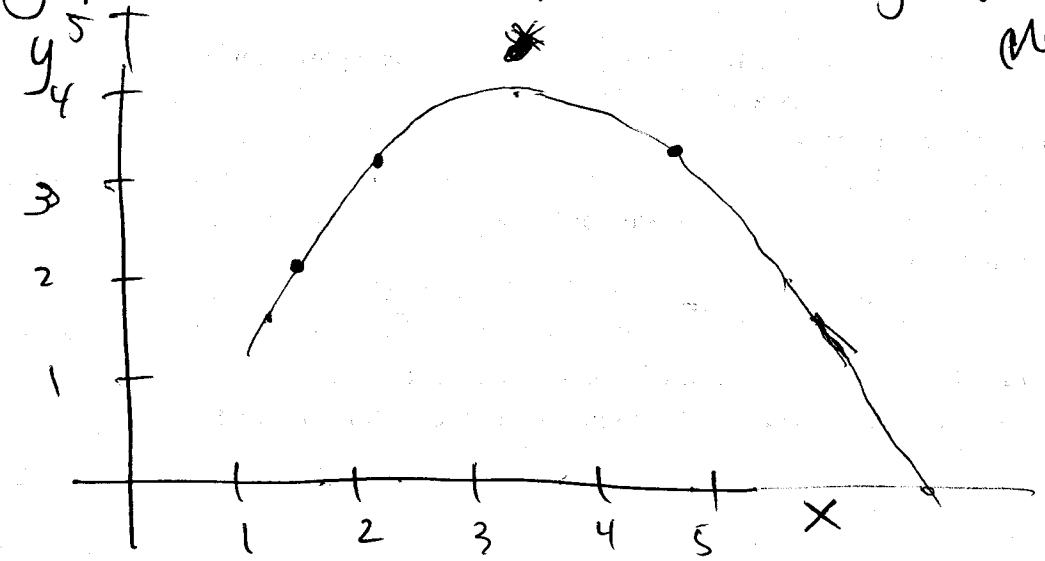
Idea: Find a curve passing through the points.

One way: use a polynomial for the curve.

Example 3.1 p24

x	1.82	2.29	4.88
y	2.03	3.28	3.73

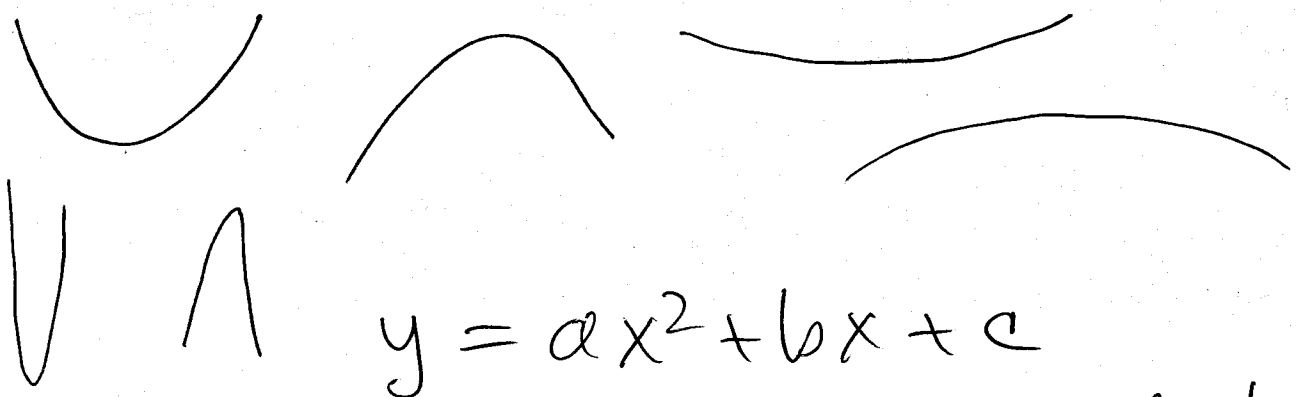
x = time in sec.
y = height in meters.



(A) Find a model for data.

From physics (and observation) we guess a quadratic might be good.

(B) Fit the data. Which quadratic?



$$y = ax^2 + bx + c$$

Find a, b, c. Need the quadratic to pass thru points.

$$a(1.82)^2 + b(1.82) + c = 2.03$$

$$a(2.29)^2 + b(2.29) + c = 3.28$$

$$a(4.88)^2 + b(4.88) + c = 3.73$$

Linear system of 3 eqns, 3 unknowns.

$$c + 1.82b + 3.3124a = 2.03$$

$$c + 2.29b + 5.2441a = 3.28$$

$$c + 4.88b + 23.8144a = 3.73$$

$$\begin{bmatrix} 1 & 1.82 & 3.3124 \\ 1 & 2.29 & 5.2441 \\ 1 & 4.88 & 23.8144 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} 2.03 \\ 3.28 \\ 3.73 \end{bmatrix}$$

Solve:

$$y = -6.1962 + 5.9984x - 0.8124x^2$$

Q: What is max height of projectile?

Q: When does it hit the ground?

$y = ax^2 + bx + c$

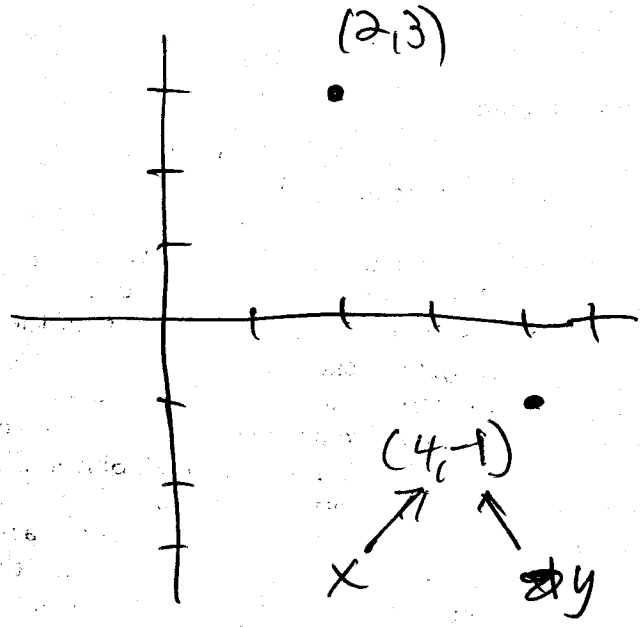
Vertex is at $x = \frac{-b}{2a}$

max height at $\frac{-5.9984}{2(-.8124)} \approx 3.69 = x$

max height: $y = -6.1962 + 5.9984(3.69) - 0.8124(3.69)^2$

≈ 4.88 meters.

eg 2.1 p18 $(2,3)$ $(4,-1)$



$y = mx + b$

$b + 2m = 3$

$b + 4m = -1$

$$-1\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -4 \end{bmatrix} \xrightarrow{\frac{1}{2}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{-2}$$

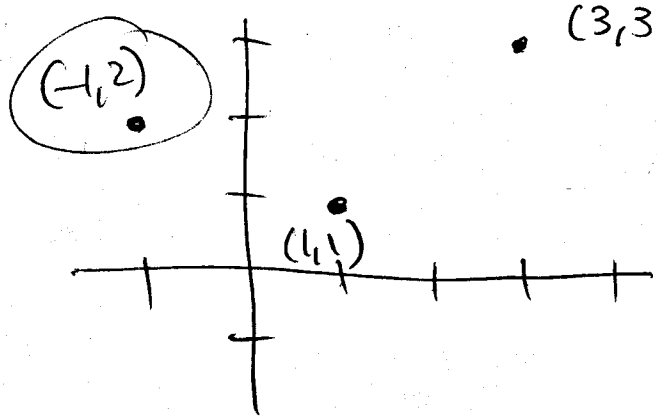
$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \end{bmatrix}$$

$m = -2$

$b = 7$

$y = -2x + 7$

e.g. Find quadratic interpolating $(-1,2)$, $(1,1)$, $(3,3)$.



$$y = \underline{c} + \underline{b}x + \underline{a}x^2$$

$$\begin{aligned} c - b + a &= 2 \\ c + b + a &= 1 \\ c + 3b + 9a &= 3 \end{aligned} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 3 \end{bmatrix}$$

$$a = \frac{3}{8} \quad b = \frac{-1}{2} \quad c = \frac{9}{8}$$

$$y = \frac{9}{8} - \frac{1}{2}x + \frac{3}{8}x^2 //$$

e.g. $(-1,-7)$, $(1,-1)$, $(4,8)$

$$y = c + bx + ax^2$$

$$\begin{aligned} c - b + a &= -7 \\ c + b + a &= -1 \\ c + 4b + 16a &= 8 \end{aligned} \rightarrow \begin{bmatrix} 1 & -1 & 1 & -7 \\ 1 & 1 & 1 & -1 \\ 1 & 4 & 16 & 8 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 1 & -1 & 1 & -7 \\ 0 & 2 & 2 & 6 \\ 0 & 5 & 15 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & -7 \\ 0 & \textcircled{1} & 1 & 3 \\ 0 & 5 & 15 & 15 \end{bmatrix}$$

$$\frac{1}{10}\begin{bmatrix} 1 & 0 & 2 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 10 & 0 \end{bmatrix} \quad -2\begin{bmatrix} 0 & 0 & 2 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

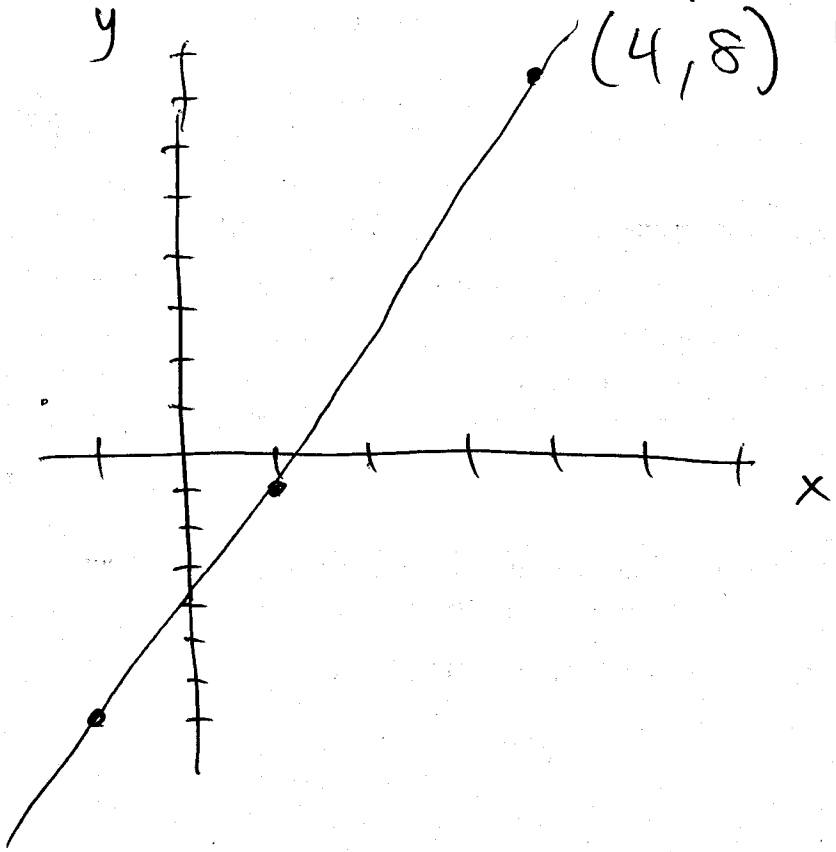
$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$c = -4$

$b = 3$

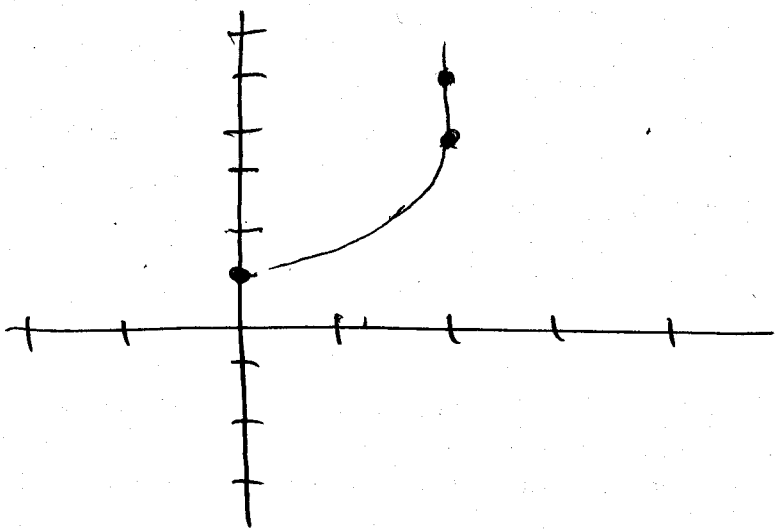
$a = 0$

$y = -4 + 3x$ so points $(-1,-7)$ $(1,-1)$ $(4,8)$ lie on a line.



e.g. $(0,1)$ $(2,4)$ $(2,5)$

Want quadratic ~~interpolant~~ interpolant.



$y = c + bx + ax^2$

Since 2 points have same x-coord, then is no interpolant.

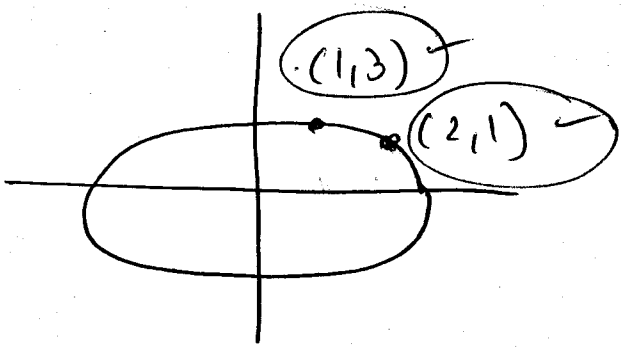
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 2 & 4 & 4 \\ 1 & 2 & 4 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$0 = 1$ no solution

Assignment

p 23 Exer 2.1 – 2.6

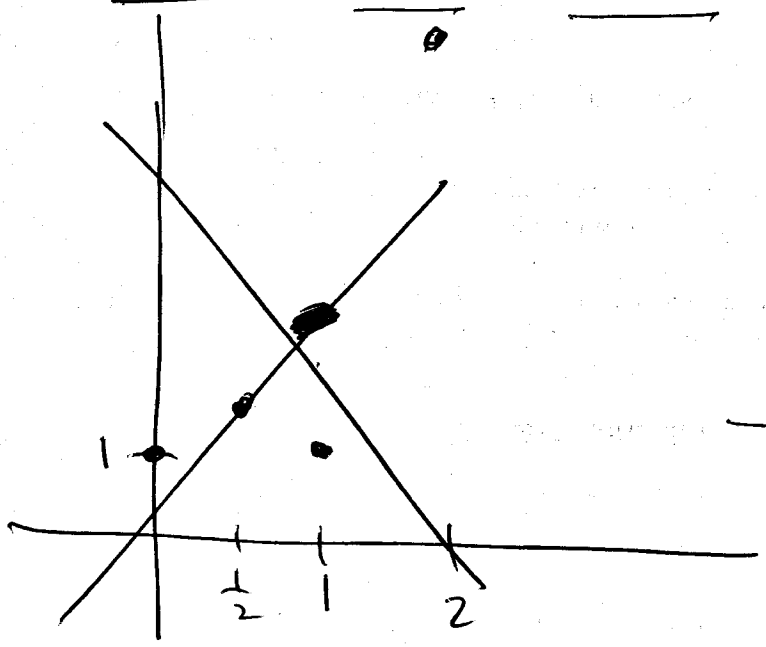
2.4 $ax^2+by^2=1$

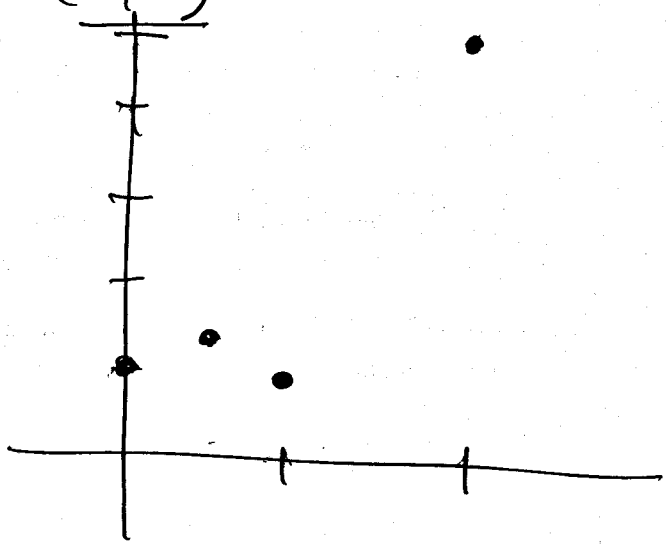


$$\begin{cases} a(1)^2+b(3)^2=1 \\ a(2)^2+b(1)^2=1 \end{cases}$$

$$\begin{aligned} a+9b&=1 \\ 4a+b&=1 \end{aligned} \rightarrow \begin{bmatrix} 1 & 9 & 1 \\ 4 & 1 & 1 \end{bmatrix}$$

2.3 $(0,1)$ $(\frac{1}{2},\frac{5}{4})$ $(1,1)$ $(2,5)$





Interpolant $y=\underline{d}+\underline{c}x+\underline{b}x^2+\underline{a}x^3$

$$d + 0 + 0 + 0 = 1$$

$$d + \frac{1}{2}c + \frac{1}{4}b + \frac{1}{8}a = \frac{5}{4}$$

$$d + c + b + a = 1$$

$$d + 2c + 4b + 8a = 2.5$$

This can be solved.

Example 3.3 p26

Cubic (degree 3) interpolant for

(30, 31.8) (40, 55.4) (50, 92.6) (60, 149.6)

MATLAB: $y = d + cx + bx^2 + ax^3$

$$\begin{bmatrix} 1 & 30 & 900 & 27000 & 31.8 \\ 1 & 40 & 1600 & 64000 & 55.4 \\ 1 & 50 & 2500 & 125000 & 92.6 \\ 1 & 60 & 3600 & 216000 & 149.6 \end{bmatrix}$$