6-4-2013

8.1 #26) 3 "states", S, M, L

current state

$$\begin{array}{c c} & \begin{array}{ccc} S & M & L \end{array} \\ \begin{array}{c} S \\ M \\ L \end{array} & \begin{bmatrix} .4 & .5 & .3 \\ .6 & 0 & .2 \\ 0 & .5 & .5 \end{bmatrix} \end{array} = A$$

next state.

$$\underline{X} = \begin{bmatrix} .8 \\ .1 \\ .1 \end{bmatrix}_0$$

initial distribution (Monday)

$$A\underline{X} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}_1$$

distribution Tuesday

$$A(A\underline{X}) = A^2\underline{X} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}_2$$

distribution Wednesday.

$$A\underline{X} = \begin{bmatrix} .4 & .5 & .3 \\ .6 & 0 & .2 \\ 0 & .5 & .5 \end{bmatrix} \begin{bmatrix} .8 \\ .1 \\ .1 \end{bmatrix} = \begin{bmatrix} .4 \\ .5 \\ .1 \end{bmatrix}_1$$

$$A^2\underline{X} = A(A\underline{X}) = \begin{bmatrix} .4 & .5 & .3 \\ .6 & 0 & .2 \\ 0 & .5 & .5 \end{bmatrix} \begin{bmatrix} .4 \\ .5 \\ .1 \end{bmatrix} = \begin{bmatrix} .44 \\ .26 \\ .3 \end{bmatrix}_2$$

44% have strenuous workout on Wed.

30) Three "states": CF CN TCT

$$\begin{array}{c} \\ CF \\ CN \\ TCT \end{array} \begin{array}{c} \begin{array}{ccc} CF & CN & TCT \end{array} \\ \begin{bmatrix} .69 & .16 & .20 \\ .12 & .74 & .14 \\ .19 & .10 & \cancel{.44}\ .66 \end{bmatrix} \end{array} = A$$

Initial distribution: $\underline{X} = \begin{bmatrix} 1500 \\ 1500 \\ 2000 \end{bmatrix}_0$

$$A\underline{X} = \begin{bmatrix} .69 & .16 & .20 \\ .12 & .74 & .14 \\ .19 & .10 & .66 \end{bmatrix} \begin{bmatrix} 1500 \\ 1500 \\ 2000 \end{bmatrix}_0 = \begin{bmatrix} 1675 \\ 1570 \\ 1755 \end{bmatrix}_1$$

1675 by CF at week 1.

$$A(A\underline{X}) = A^2\underline{X} = \begin{bmatrix} .69 & .16 & .20 \\ .12 & .74 & .14 \\ .19 & .10 & .66 \end{bmatrix} \begin{bmatrix} 1675 \\ 1570 \\ 1755 \end{bmatrix}_1 = \begin{bmatrix} 1758 \\ 1608.5 \\ 1633.6 \end{bmatrix}_2$$

About 1634 by TCT at week 2.

MyMathLab Quiz 3 Problem 1

$$\begin{array}{c|c} & \begin{array}{ccc} C & E & S \end{array} \\ \begin{array}{c} C \\ E \\ S \end{array} & \begin{bmatrix} .03 & .02 & .10 \\ .20 & .02 & 0 \\ .30 & .02 & .04 \end{bmatrix} \end{array} = A \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(I - A)X = \begin{bmatrix} .97 & -.02 & -.10 \\ -.20 & .98 & 0 \\ -.30 & -.02 & .96 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$(I - A) \qquad X = D$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \Big[$$

Found this using MATLAB

Problem 2

$$A = \begin{bmatrix} .28 & .41 \\ .36 & .18 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .28 & .41 \\ .36 & .18 \end{bmatrix} = \begin{bmatrix} .72 & -.41 \\ -.36 & .82 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{.4428} \begin{bmatrix} .82 & .41 \\ .36 & .72 \end{bmatrix} \cong \begin{bmatrix} 1.85 & .93 \\ .81 & 1.63 \end{bmatrix}$$

$$D = (.72)(.82) - (.36)(.41)$$

$$= .4428$$

$$\begin{bmatrix} 1.85 & .93 \\ .81 & 1.63 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 13 \\ 13 \end{bmatrix}$$

Problem 5

$$\begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix} \begin{bmatrix} .47 \\ .53 \end{bmatrix}_0 \cong \begin{bmatrix} .54 \\ .46 \end{bmatrix}_1$$

$$\begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix} \begin{bmatrix} .54 \\ .46 \end{bmatrix}_1 \cong \begin{bmatrix} .56 \\ .44 \end{bmatrix}_2$$

$$A^2 = \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix} \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix} = \begin{bmatrix} .61 & .52 \\ .39 & .48 \end{bmatrix}$$

$$A^2 \begin{bmatrix} .47 \\ .53 \end{bmatrix} = \begin{bmatrix} .61 & .52 \\ .39 & .48 \end{bmatrix} \begin{bmatrix} .47 \\ .53 \end{bmatrix}_0 \cong \begin{bmatrix} .56 \\ .44 \end{bmatrix}_2$$

Problem 7

(a)
$$\begin{array}{c|cc} & L & R \\ \hline L & & \\ R & & \end{array}\quad \begin{array}{cc} & \\ L & \left[\begin{array}{cc} .9 & .6 \\ .1 & .4 \end{array}\right] = A \\ R & \end{array}$$

(b) $$A^2 = \begin{bmatrix} .9 & .6 \\ .1 & .4 \end{bmatrix} \begin{bmatrix} .9 & .6 \\ .1 & .4 \end{bmatrix} = \begin{bmatrix} .87 & .78 \\ .13 & .22 \end{bmatrix}$$

(c) $$\begin{bmatrix} .9 & .6 \\ .1 & .4 \end{bmatrix} \begin{bmatrix} .5 \\ .5 \end{bmatrix}_0 = \begin{bmatrix} .75 \\ .25 \end{bmatrix}_1$$

$$\begin{bmatrix} .9 & .6 \\ .1 & .4 \end{bmatrix} \begin{bmatrix} .75 \\ .25 \end{bmatrix}_1 = \begin{bmatrix} .825 \\ .175 \end{bmatrix}_2$$

(d) 85.7%

8.2 Regular Stochastic Matrices.

Idea: with our examples, we notice that $A, A^2, A^3, A^4, \ldots, A^{100}, \ldots$ begin to resemble a single matrix with all columns the same.

Q: Does this always happen?

Q: What does it mean?

Q: Can we find this matrix without tons of computation?

Does this always happen? NO

A must be a regular stochastic matrix, i.e., some power of A has all positive entries.

e.g. $\begin{bmatrix} .6 & .6 & .6 \\ .3 & .3 & .3 \\ .1 & .1 & .1 \end{bmatrix}$ regular? YES

e.g. $\begin{bmatrix} .4 & 1 \\ .6 & 0 \end{bmatrix}$ regular? MAYBE

$$\begin{bmatrix} .4 & 1 \\ .6 & 0 \end{bmatrix}^2 = \begin{bmatrix} .4 & 1 \\ .6 & 0 \end{bmatrix}\begin{bmatrix} .4 & 1 \\ .6 & 0 \end{bmatrix} = \begin{bmatrix} .76 & .4 \\ .24 & .6 \end{bmatrix}$$

$\therefore$ A is <u>regular</u>.

<u>eg</u> $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ regular?

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Powers of A just oscillate between $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

<u>So not regular</u>.

What does it mean?

① The columns of this matrix (where all columns are the same) gives the stable distribution of A.

② No matter what initial distribution you have, eventually you will arrive at the stable distribution.

How do we find it?

Idea: If the distribution $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is stable, then we expect

$$AX = X$$