6-3-2013

Matlab Assignment 1 due Wed by 9:30

Please submit only one copy of group work.

Continuing with 2.6 (Input/Output).

Example 1   Input requirements (to produce \$1 of output).

$$\text{From}\quad \begin{array}{c|c} & \begin{array}{ccc} C & S & E \end{array} \\ \hline \begin{array}{c} C \\ S \\ E \end{array} & \begin{bmatrix} 0 & .15 & .43 \\ .02 & .03 & \boxed{.20} \\ .01 & .08 & .05 \end{bmatrix} \end{array}$$

To produce \$1 worth of E, require \$.20 worth of S.

Each industry can set its production level.

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Q: How much is left over for consumers?

Why is it not $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?

Because each industry consumes some output of itself and other two.

How much is consumed internally to produce $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?

$$
\underbrace{\begin{bmatrix} 0 & .15 & .43 \\ .02 & .03 & .20 \\ .01 & .08 & .05 \end{bmatrix}}_{A}
\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\underline{X}}
=
\begin{array}{cl}
\begin{bmatrix} 0 \cdot x + (.15)y + (.43)z \\ (.02)x + (.03)y + (.20)z \\ (.01)x + (.08)y + (.05)z \end{bmatrix} &
\begin{array}{l} \leftarrow C \\ \leftarrow S \\ \leftarrow E \end{array}
\end{array}
$$

So how much is left over for consumers?

$$
\begin{bmatrix} x - [(0)x + (.15)y + (.43)z] \\ y - [(.02)x + (.03)y + (.20)z] \\ z - [(.01)x + (.08)y + (.05)z] \end{bmatrix} = \underline{X} - A\underline{X}
$$

But $\underline{X} - A\underline{X} = (I - A)\underline{X}$.

Given a <u>final-demand</u> matrix D, what should production level be?

$$
D = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}. \longrightarrow \text{Solve } \underline{X} - A\underline{X} = D
$$

$$
(I - A)\underline{X} = D
$$

$$
\underline{X} = (I - A)^{-1} D.
$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & .15 & .43 \\ .02 & .03 & .20 \\ .01 & .08 & .05 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & -.15 & -.43 \\ -.02 & .97 & -.20 \\ -.01 & -.08 & .95 \end{bmatrix}$$

$$\underline{X} = (I-A)^{-1} D = \begin{bmatrix} 1 & -.15 & -.43 \\ -.02 & .97 & -.20 \\ -.01 & -.08 & .95 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1.01 & .20 & .50 \\ .02 & 1.05 & .23 \\ .01 & .09 & 1.08 \end{bmatrix}}_{\text{this is given.}} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3.72 \\ 1.78 \\ 3.35 \end{bmatrix}$$

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Input requirements

$$\begin{array}{c c} & \begin{array}{ccc} C & S & B \end{array} \\ \text{From} \begin{array}{c} C \\ S \\ B \end{array} & \begin{bmatrix} .02 & .02 & .10 \\ .20 & .01 & 0 \\ .10 & .02 & .01 \end{bmatrix} \end{array} = A \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Final-demand matrix $D = \begin{bmatrix} 4.5 \\ 2 \\ 1 \end{bmatrix}$

$$\underline{X} = (I - A)^{-1} D$$

$$I - A = \begin{bmatrix} .98 & -.02 & -.10 \\ -.20 & .99 & 0 \\ -.10 & -.02 & .99 \end{bmatrix}$$

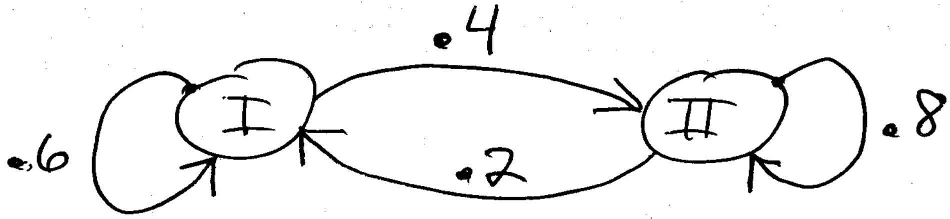
$$(I - A)^{-1} = \begin{bmatrix} 1.04 & .02 & .10 \\ .21 & 1.01 & .02 \\ .11 & .02 & 1.02 \end{bmatrix}$$

$$\underline{X} = (I - A)^{-1} \begin{bmatrix} 4.5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.82 \\ 2.99 \\ 1.56 \end{bmatrix}$$

Markov Processes.

Idea: A Markov process is a good model for certain situations.

Example: Spy located in one of 2 cities



Suppose we know spy was in city I 3 days ago. Where is she likely to be today?

Know: transition probabilities.

current location

$$A = \begin{array}{c|cc} & \text{I} & \text{II} \\ \hline \text{I} & .6 & .2 \\ \text{II} & .4 & .8 \end{array}$$

next location

Initial location is I.

Let $X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

day 1: $AX = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .6 \\ .4 \end{bmatrix}$

day 2: $A(AX) = A^2X = \underbrace{\begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} .6 \\ .4 \end{bmatrix}}_{AX} = \begin{bmatrix} .44 \\ .56 \end{bmatrix}$

day 3:

$$A(A^2X) = A^3 X = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \begin{bmatrix} .44 \\ .56 \end{bmatrix} = \begin{bmatrix} .376 \\ .624 \end{bmatrix}$$

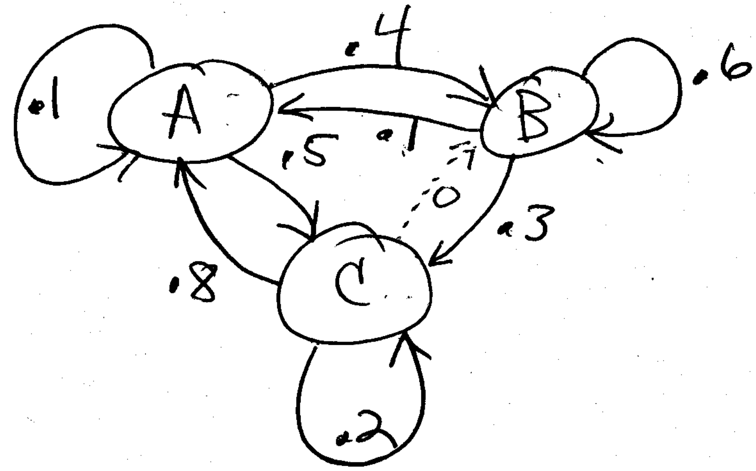
Note: We need powers of A.

$$A = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} = \begin{bmatrix} .44 & .28 \\ .56 & .72 \end{bmatrix}.$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \begin{bmatrix} .44 & .28 \\ .56 & .72 \end{bmatrix} = \begin{bmatrix} .376 & .312 \\ .624 & .688 \end{bmatrix}$$

Note: Sum of columns of A are ~~the~~ 1 and also sum of columns of $A^2, A^3, \ldots$ are 1. Such a matrix is a stochastic matrix, i.e. ① all entries are ≥ 0 and ② columns add to 1, ③ matrix is square eg 10.



$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} .1 & .1 & .8 \\ .4 & .6 & 0 \\ .5 & .3 & .2 \end{bmatrix} \end{array}$$

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$$\begin{array}{cc} & \begin{array}{cc} D & R \end{array} \\ \begin{array}{c} D \\ R \end{array} & \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix} \end{array} = A$$

$$A^2 = \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix} = \begin{bmatrix} .61 & .52 \\ .39 & .48 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .61 & .52 \\ .39 & .48 \end{bmatrix} = \begin{bmatrix} .583 & .556 \\ .417 & .444 \end{bmatrix}$$