

2.6 Input/Output Analysis

Idea: Use linear methods to model an economy.

Example 1

		Input ←	requirements to produce \$1 worth of each <u>column</u> .	
		C	S	E
From	C	0	.15	.43
	S	.02	.03	.20
	E	.01	.08	.05

Suppose each industry produces at a level of  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ , i.e.  $x$  billion \$ of coal,  $y$  billion \$ of Steel,  $z$  billion \$ of electricity. However,  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  is not available to consumers because some of it is used by industries. How much?

$$\begin{array}{c}
 C \\
 S \\
 E
 \end{array}
 \begin{bmatrix}
 0 & .15 & .43 \\
 .02 & .03 & .20 \\
 .01 & .08 & .05
 \end{bmatrix}
 \begin{bmatrix}
 x \\
 y \\
 z
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \cdot x + .15y + .43z \\
 .02x + .03y + .20z \\
 .01x + .08y + .05z
 \end{bmatrix}$$

$A \quad X = AX$

So  $AX$  is being used by industries  
in production.

$X - AX$  is available to consumers.

$$X - AX = (I - A)X$$

Solve  $X - AX = D$

$$(I - A)X = D$$

$$X = (I - A)^{-1}D.$$