Exam 2 2.1 – 2.4 5-29-2013

2.3/2.4 Recap

Linear system looks like simple equation

$ax = b \longrightarrow x = \frac{b}{a}$.

A linear system of n equations in n unknowns can be written

$$\underset{\substack{n\times n\\ \text{matrix}}}{A}\;\underset{\substack{n\times 1\\ \text{matrix}}}{X} = b$$

$$4x - 2y = 5$$
$$3x + y = 1$$

$$\underset{2\times 2}{\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 5 \\ 1 \end{bmatrix}}$$

Hope is that matrices can be treated like numbers.

① Adding matrices.

a) must be same size

b) add elements entry by entry.

e.g.
$$\begin{bmatrix} 4 & -2 \\ 3 & 1 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4+2 & -2+0 \\ 3+1 & 1+0 \\ 5+1 & 1+3 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 4 & 1 \\ 6 & 4 \end{bmatrix}$$

② Multiplying matrices.

a) sizes must work out

b) columns of second matrix multiplied by rows of first matrix.

c) <u>not</u> <u>commutative</u> $\quad 2\cdot 3 = 3\cdot 2$

e.g.
$$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 5 & 8 \end{bmatrix}$$

$2\times 2 \qquad 2\times 2$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 20 & 1 \end{bmatrix}$$

e.g.
$$\begin{bmatrix} 2 & 8 \\ -1 & 5 \\ 1 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} 6 & 8 & 2 \\ 1 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 20 & 32 & -20 \\ -1 & 2 & -17 \\ 8 & 12 & -4 \\ 45 & 62 & 5 \end{bmatrix}$$

$4\times 2 \qquad 2\times 3 \qquad\qquad 4\times 3$

③ Multiply matrix by a number.

a) matrix can be any size

b) multiply each entry by the number.

e.g.

$$\left(\tfrac{1}{2}\right)\cdot\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -\tfrac{1}{2} \\ 1 & \tfrac{1}{2} \end{bmatrix}$$

eg #50 p83) (a)

$$AC = \begin{array}{c} 20 \quad\;\; 10 \quad\;\; 35 \\ \cancel{40} \quad \cancel{30} \quad \cancel{15} \\ \begin{bmatrix} 300 & 250 & 450 \end{bmatrix} \end{array} \begin{bmatrix} 30 & 40 & 20 \\ 20 & 30 & 10 \\ 10 & 5 & 35 \end{bmatrix}$$

1×3 $\qquad$ 3×3

$$= \begin{bmatrix} 18500 & 21750 & 24250 \end{bmatrix}$$

total wholesale sales revenue from each store in September

(b) AD = same as (a) from October.

~~(c)~~ $AC + AD \rightarrow$ total wholesale sales from Sept and October.

$$\boxed{AC + AD = A(C + D)}$$

Inverses

Identity matrix: I or I_n

is an $n \times n$ matrix

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 \cdots & 0 \\ & & \ddots & \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

eg

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$$

Inverse of a matrix: Given A an $n \times n$ matrix (so square), the inverse A^{-1} satisfies

$$AA^{-1} = I \text{ and } A^{-1}A = I.$$

Only square matrices have inverses

We have a formula for 2x2 matrices.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$D = ad - bc$. (If $D=0$ then matrix has no inverse.)

Using inverses:

write a linear system as $AX = b$

A: $n \times n$, X: $n \times 1$, b: $n \times 1$

$$\underbrace{A^{-1}A}_{I}X = A^{-1}b$$

$$X = A^{-1}b.$$

e.g. 6) p93 $A = \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$

Solve
$$\begin{aligned} 4x - 2y + 3z &= 1 \\ 8x - 3y + 5z &= 4 \\ 7x - 2y + 4z &= 5 \end{aligned}$$

$$\begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$$

$$A \quad X = b$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} b = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -9 \end{bmatrix}$$

$\underline{x = 4 \quad y = -6 \quad z = -9}$

$$\frac{1}{4} \left[\begin{array}{cc|c} 3 & -\frac{1}{4} & 2 \\ 0 & 1 & 8 \end{array}\right] \longrightarrow \begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 8 \end{bmatrix}$$

$$\begin{matrix} 3x - \frac{1}{4}y = 2 \\ y = 8 \end{matrix} \longrightarrow \begin{matrix} 3x = 4 \\ y = 8 \end{matrix} \longrightarrow \begin{matrix} x = \frac{4}{3} \\ y = 8 \end{matrix}$$

eg

$$\begin{aligned} 3x - 4y \quad &= 7 \\ 8x + 7y - 6z &= 0 \\ -6x \quad + 6z &= 3 \end{aligned} \rightarrow \frac{1}{3}\begin{bmatrix} 3 & -4 & 0 & 7 \\ 8 & 7 & -6 & 0 \\ -6 & 0 & 6 & 3 \end{bmatrix}$$

$$6\cancel{8}\begin{bmatrix} 1 & -\frac{4}{3} & 0 & \frac{7}{3} \\ 8 & 7 & -6 & 0 \\ -6 & 0 & 6 & 3 \end{bmatrix} \rightarrow \frac{3}{53}\begin{bmatrix} 1 & -\frac{4}{3} & 0 & \frac{7}{3} \\ 0 & \frac{53}{3} & -6 & -\frac{56}{3} \\ 0 & -8 & 6 & 17 \end{bmatrix}$$

$$\frac{32}{3} + \frac{21}{3} = \frac{53}{3} \qquad 8\,\cancel{\tfrac{4}{8}}\begin{bmatrix} 1 & -\frac{4}{3} & 0 & \frac{7}{3} \\ 0 & 1 & \frac{-18}{53} & -\frac{56}{53} \\ 0 & -8 & 6 & 17 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{-24}{53} & \cancel{}\,* \\ 0 & 1 & -\frac{18}{53} & -\frac{56}{53} \\ 0 & 0 & \left(-\frac{18}{53}(8) + 6\right) & \end{bmatrix} \leftarrow \left(-\frac{56}{53}(8) + 17\right)$$

Help!

$$\frac{4}{\cancel{3}} \cdot \frac{-\cancel{18}\,6}{53} = \frac{-24}{53}$$

$$\cancel{\frac{4}{3}\left(-\frac{56}{53}\right) + \frac{7}{3} = -\frac{4 \cdot 56}{9} + \frac{21}{9} = \frac{-203}{9}}$$

eg

$$\begin{array}{r} \\ \\ -5\cdot\cancel{3} \end{array}\begin{bmatrix} 1 & 0 & 3 & 8 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$x=-4 \quad y=-13 \quad z=4$

eg

$$\begin{array}{r} -1 \\ \\ \\ \end{array}\begin{bmatrix} -1 & 1 & 1 & -2 \\ -1 & 5 & -23 & -14 \\ 3 & -1 & -15 & 0 \end{bmatrix} \quad \begin{array}{r} -3 \\ \\ \\ \end{array}\begin{bmatrix} 1 & -1 & -1 & 2 \\ -1 & 5 & -23 & -14 \\ 3 & -1 & -15 & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{4} \\ \\ \end{array}\begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 4 & -24 & -12 \\ 0 & 2 & -12 & -6 \end{bmatrix} \quad \begin{array}{r} \\ -2 \\ \\ \end{array}\begin{bmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & -6 & -3 \\ 0 & 2 & -12 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -7 & -1 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x \quad -7z=-1$

$y \quad -6z=-3$

$\uparrow$

$x = 7z-1$

$y = 6z-3$

$z =$ any value

eg

$$\begin{bmatrix} 6000 & 5000 & 7000 \end{bmatrix} \begin{bmatrix} .25 & .75 \\ .65 & .35 \\ .51 & .49 \end{bmatrix}$$

$$= \begin{bmatrix} 8320 & 9680 \end{bmatrix}$$