5-28-2013

Recap: Solving systems of linear equations

Gauss-Jordan Method

Elementary Row operations

Easier to work with the matrix of coefficient (the augmented matrix)

e.g.,

$$\begin{aligned} x+2y&=1 \\ -5x+4y&=1 \end{aligned} \longrightarrow \begin{array}{c} 5 \\ \end{array}\left[\begin{array}{cc|c} 1 & 2 & 1 \\ -5 & 4 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \frac{1}{14} \end{array}\left[\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 14 & 6 \end{array}\right] \rightarrow \begin{array}{c} \\ -2 \end{array}\left[\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 1 & \frac{3}{7} \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{1}{7} \\ 0 & 1 & \frac{3}{7} \end{array}\right] \quad \begin{aligned} x&=\frac{1}{7} \\ y&=\frac{3}{7} \end{aligned}$$

$$-2\left(\frac{3}{7}\right)+1=\frac{-6}{7}+\frac{7}{7}=\frac{1}{7}$$

Q: What does it mean when you can't pivot?

You have more than 1 or no solutions.

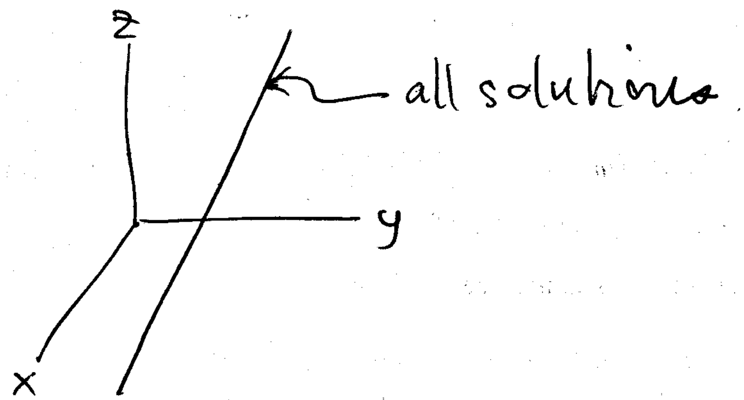
e.g. 14)
p69

$$\begin{aligned} x - 3y + z &= 5 \\ -2x + 7y - 6z &= -9 \\ x - 2y - 3z &= 6 \end{aligned} \longrightarrow \begin{array}{c} -1x \\ \\ \\ \end{array}\begin{bmatrix} 1 & -3 & 1 & 5 \\ -2 & 7 & -6 & -9 \\ 1 & -2 & -3 & 6 \end{bmatrix}$$

$$\begin{array}{c} \\ -1x \\ \\ \end{array}\begin{bmatrix} 1 & -3 & 1 & 5 \\ 0 & 1 & -4 & 1 \\ 0 & 1 & -4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -11 & 8 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x \quad - 11z &= 8 \\ y - 4z &= 1 \\ 0 &= 0 \end{aligned} \qquad\qquad \begin{aligned} x &= 11z + 8 \\ y &= 4z + 1 \\ z &= \text{any value} \end{aligned}$$

This system has a <u>line</u> of solutions



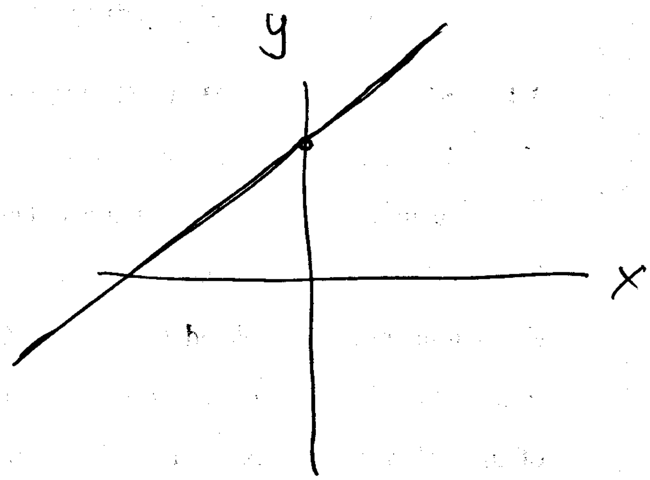
12)

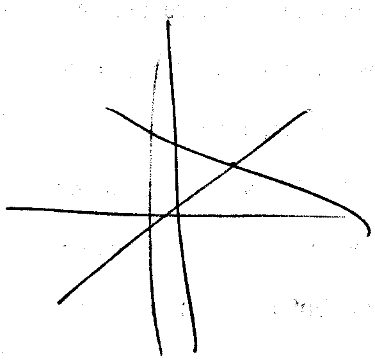
$$\begin{aligned} x - 6y &= 12 \\ -\tfrac{1}{2}x + 3y &= -6 \\ \tfrac{1}{3}x - 2y &= 4 \end{aligned} \longrightarrow \begin{array}{c} -\frac{1}{3} \\ \cancel{\frac{1}{2}} \\ \\ \end{array} \begin{bmatrix} 1 & -6 & 12 \\ -\frac{1}{2} & 3 & -6 \\ \frac{1}{3} & -2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -6 & 12 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{aligned} x - 6y &= 12 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

$x = 6y + 12$

$y =$ any value





#30) x = # of Type I plants
y = " " II "
z = " " III "

$$\begin{aligned} 7x + 10y + 13z &= 150 \\ x + y + z &= 15 \end{aligned} \quad \rightarrow \quad \begin{array}{c} -7 \\ \\ \end{array}\begin{bmatrix} 1 & 1 & 1 & 15 \\ 7 & 10 & 13 & 150 \end{bmatrix}$$

$$\begin{array}{c} \\ \frac{1}{3} \end{array}\begin{bmatrix} 1 & 1 & 1 & 15 \\ 0 & 3 & 6 & 45 \end{bmatrix} \rightarrow \begin{array}{c} \\ -1 \end{array}\begin{bmatrix} 1 & 1 & 1 & 15 \\ 0 & 1 & 2 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 15 \end{bmatrix} \qquad \begin{aligned} x \quad - z &= 0 \\ y + 2z &= 15 \end{aligned}$$

$x = z$
$y = -2z + 15$
z = any value

2.3 Arithmetic Operations on Matrices.

Idea: Solve: $\frac{1}{3}(3\cdot x) = 7 \cdot \frac{1}{3} \longrightarrow \quad x = \frac{7}{3}$.

This is very easy. Does this idea extend to systems?

$$5x - 3y = \frac{1}{2}$$
$$4x + 2y = -1$$

unknown $= \begin{bmatrix} x \\ y \end{bmatrix}$ 2×1 matrix

right side $= \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix}$ 2×1 matrix

coefficients $= \begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}$ 2×2 matrix

write system as:

$$\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix}$$

what replaces multiplication? Matrix multiplication

How do you do a matrix product?

eg (2×2) (2×1)

$$\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{array}{c} [x \quad y] \\ \begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 5x - 3y \\ 4x + 2y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix}.$$

eg

$$\begin{bmatrix}2&1\\0&1\\1&0\end{bmatrix}\begin{bmatrix}1&1\\4&2\end{bmatrix}=\begin{bmatrix}6&4\\4&2\\1&1\end{bmatrix}$$

$3\times\cancel{2}$ $\cancel{2}\times 2$ 3×2

match

$$\begin{array}{cc} 1 & 2 \\ & 4 \\ \multicolumn{2}{c}{\begin{bmatrix}2&1\\0&1\\1&0\end{bmatrix}} \end{array} \rightarrow 2\cdot1+1\cdot4=6$$

$$\begin{array}{cc} 1 & 4 \\ \multicolumn{2}{c}{\begin{bmatrix}2&1\end{bmatrix}} \end{array}\begin{bmatrix}1\\4\end{bmatrix}=2\cdot1+4\cdot1$$

eg

$$\begin{array}{ccc} 1 & 5 & 2 \\ \multicolumn{3}{c}{\begin{bmatrix}3&-1\\2&0\\1&5\end{bmatrix}\begin{bmatrix}1&0\\5&-4\\2&-1\end{bmatrix}} \end{array}$$ ~~is~~ is not defined

3×2 3×2

2.4 The inverse of a matrix

$$3x = 7 \rightarrow (3^{-1}\cdot 3)x = 3^{-1}\cdot 7$$

$$x = \frac{7}{3}.$$

$$\begin{aligned} 5x - 3y &= \tfrac{1}{2} \\ 4x + 2y &= -1 \end{aligned} \longrightarrow \begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \tfrac{1}{2} \\ -1 \end{bmatrix}$$

$$A \quad X = B$$

$$\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 0 \\ 0 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 22 & 0 \\ 0 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} \tfrac{1}{2} \\ -1 \end{bmatrix}$$

$$\boxed{\begin{aligned} x &= \frac{-1}{11} \\ y &= \frac{-7}{22} \end{aligned}}$$

$$\begin{bmatrix} 22 & 0 \\ 0 & 22 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -7 \end{bmatrix} \longrightarrow \begin{aligned} 22x &= -2 \\ 22y &= -7 \end{aligned}$$

In fact:

$$\frac{1}{22}\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{11} & \frac{3}{22} \\ -\frac{2}{11} & \frac{5}{22} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{11} & \frac{3}{22} \\ \frac{-2}{11} & \frac{5}{22} \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} \frac{1}{11} & \frac{3}{22} \\ \frac{-2}{11} & \frac{5}{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We say: $\begin{bmatrix} \frac{1}{11} & \frac{3}{22} \\ \frac{-2}{11} & \frac{5}{22} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}^{-1}$

Formula for 2×2 inverse:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

where $D = ad - bc$ (the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.)

Then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

#6) Find $\begin{bmatrix} 1 & .5 \\ 0 & .5 \end{bmatrix}^{-1} = \frac{1}{.5}\begin{bmatrix} .5 & -.5 \\ 0 & 1 \end{bmatrix}$

$D = (1)(.5) - (.5)(0) = .5$

$= 2\begin{bmatrix} .5 & -.5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & .5 \\ 0 & .5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(0)(-1) + (.5)(2) = 1$