5-23-2013

No class Monday 5/27
Exams returned Tuesday 5/28
Grades available by email or Blackboard

Solving systems of linear equations
Gauss-Jordan method
Applying elementary <u>row</u> operations

<u>e.g.</u>

$$3x - 2y = 1$$
$$2x + y = 10$$

<u>Main observation</u>: only the coefficients matter, not the variables.
Only need to look at matrix:

$$\begin{array}{c} \frac{1}{3} \\ \end{array}\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 1 & 10 \end{array}\right]\begin{array}{l} \leftarrow \text{row} \\ \leftarrow \text{row} \end{array}$$

column

$$\downarrow$$

$$\begin{array}{c} -2\cdot \\ \end{array}\left[\begin{array}{cc|c} 1 & -\frac{2}{3} & \frac{1}{3} \\ 2 & 1 & 10 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} \\ \frac{3}{7} \end{array}\left[\begin{array}{cc|c} 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & \frac{7}{3} & \frac{28}{3} \end{array}\right]$$

$$\rightarrow$$

$$\begin{array}{c} \\ \frac{2}{3} \end{array}\left[\begin{array}{cc|c} 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 4 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \end{array}\right]$$

$$\underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{\text{diagonal form}} \quad \underbrace{\begin{matrix} 3 \\ 4 \end{matrix}}_{\text{solution}}$$

$x = 3$
$y = 4$

e.g.

$$y + 3z = 1$$
$$x + 6y - 4z = 1$$
$$-3y + 7z = 2$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 1 \\ 1 & 6 & -4 & 1 \\ 0 & -3 & 7 & 2 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} \\ -6/3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & -3 & 7 & 2 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} \\ \\ \frac{1}{16} \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -22 & -5 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 16 & 5 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} \\ \\ 22/-3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -22 & -5 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & \frac{5}{16} \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{15}{8} \\ 0 & 1 & 0 & \frac{1}{16} \\ 0 & 0 & 1 & \frac{5}{16} \end{array}\right]$$

$$\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\text{diagonal form}} \quad \underbrace{\begin{matrix} \frac{15}{8} \\ \frac{1}{16} \\ \frac{5}{16} \end{matrix}}_{\text{solution}}$$

$$x = \frac{15}{8}$$
$$y = \frac{1}{16}$$
$$z = \frac{5}{16}.$$

$$\frac{110}{16} - \frac{80}{16} = \frac{30}{16} = \frac{15}{8}$$

eg #56)
p 62

x = # bottles of national brand
y = # bottles of store brand

$$x + y = 82$$

$$1.99x + 1.79y = 158.98$$

$$\begin{array}{r c} -1.99 & \left[\begin{array}{cc|c} 1 & 1 & 82 \\ 1.99 & 1.79 & 158.98 \end{array}\right] \end{array}$$

$$\downarrow$$

$$\begin{array}{r c} & \left[\begin{array}{ccc} 1 & 1 & 82 \\ 0 & -.2 & -4.2 \end{array}\right] \\ -5 & \end{array}$$

$$\downarrow$$

$$\begin{array}{r c} & \left[\begin{array}{ccc} 1 & 1 & 82 \\ 0 & 1 & 21 \end{array}\right] \\ -1 & \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc} 1 & 0 & 61 \\ 0 & 1 & 21 \end{array}\right]$$

$x = 61$
$y = 21$

#60) $x = \#$ oz. of I

$y = \#$ oz. of II

$z = \#$ oz. of III.

$$.10x + .10y + .10z = 1 \leftarrow \text{carb}$$

$$.10x + .05y + .25z = 1 \leftarrow \text{protein}$$

$$.15x + .10z = 1 \leftarrow \text{vit. c}$$

$$\left[\begin{array}{ccc|c} .1 & .1 & .1 & 1 \\ .1 & .05 & .25 & 1 \\ .15 & 0 & .1 & 1 \end{array}\right]$$

etc...

2.2 Solving Systems, II.

① Pivoting.

a. pick an entry in matrix

b. make the entry 1

c. eliminate remaining entries in that column.

eg. #2)

$$\begin{array}{c} \\ \frac{1}{8} \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & -12 \end{bmatrix}$$

$$\downarrow$$

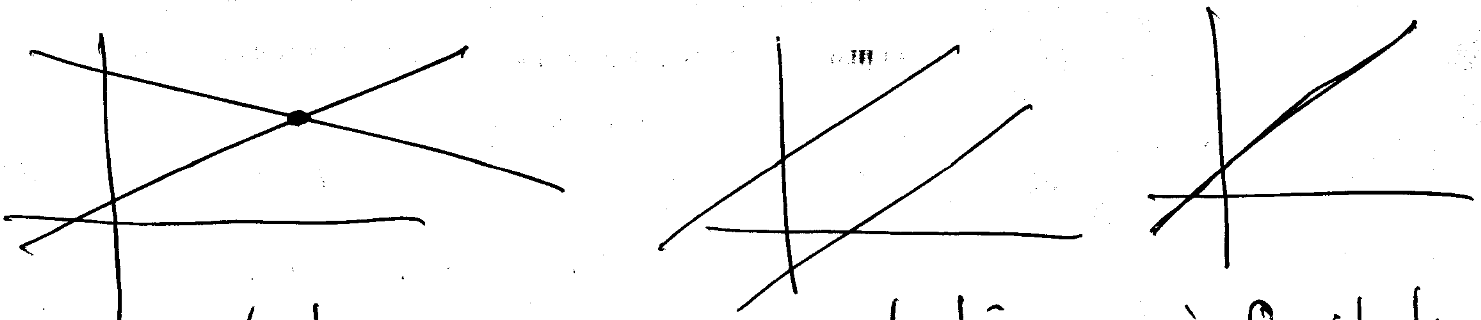
$$\begin{array}{c} \\ -2 \end{array} \begin{bmatrix} 1 & 2 & 3 \\ \frac{1}{2} & 1 & -\frac{3}{2} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 1 & -\frac{3}{2} \end{bmatrix}$$

② Multiple solutions / No solutions

2 equns
2 unknowns : $\quad 3x - 2y = 1$
$\qquad\qquad\qquad\;\; 2x + y = 10$



one solution $\qquad$ no solution $\qquad$ infinitely many soln.

Q: What does Gauss-Jordan look like if we have no solution or many solutions?

e.g.

$$-\tfrac{1}{2}x + y = \tfrac{3}{2}$$
$$-3x + 6y = 10$$

$$-2\left[\begin{array}{cc|c} -\frac{1}{2} & 1 & \frac{3}{2} \\ -3 & 6 & 10 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & -6 \\ -3 & 6 & 10 \end{array}\right]$$

$$\to \left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & 0 & -8 \end{array}\right] \qquad \begin{aligned} x - 2y &= -6 \\ 0 &= -8 \end{aligned}$$

stuck. $\qquad \boxed{\text{no solutions}}$

$-\frac{1}{2}x + y = \frac{3}{2} \longrightarrow y = \frac{1}{2}x + \frac{3}{2}$

$-3x + 6y = 10 \longrightarrow 6y = 3x + 10$

$y = \frac{1}{2}x + \frac{5}{3}$

lines are parallel

e.g

$$2x + 4y = 3$$
$$-x - 2y = -\frac{3}{2}$$

$$\frac{1}{2}\left[\begin{array}{cc|c} 2 & 4 & 3 \\ -1 & -2 & -\frac{3}{2} \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & \frac{3}{2} \\ -1 & -2 & -\frac{3}{2} \end{array}\right]$$

$$\rightarrow \begin{array}{cc} \left[\begin{array}{cc|c} 1 & 2 & \frac{3}{2} \\ 0 & 0 & 0 \end{array}\right] \\ \quad x \quad\; y \end{array}$$

$x + 2y = \frac{3}{2}$

$0 = 0$

infinitely many solutions.

y = any value

$x = -2y + \frac{3}{2}$

Every value I pick for y gives me a solution

$y = 0 \quad x = \frac{3}{2}$ ← solution

$y = 2 \quad x = -4 + \frac{3}{2} = -\frac{5}{2}$ ← another solution