5-22-2013

Exam 1, Thursday 1.1, 1.3, 1.4

Solving systems of linear equations.

Gauss-Jordan Method

- elementary row operations
- idea: Manipulate system without changing the solutions.

e.g. $\frac{1}{3}(3x - 2y = 1)$ $\leftarrow \frac{1}{3}R_1$

$2x + y = 10$

$\downarrow$

$-2(x - \frac{2}{3}y = \frac{1}{3})$

$2x + y = 10$ $\leftarrow -2R_1 + R_2$

$\downarrow$

$\frac{4}{3}y + \frac{3}{3}y = \frac{7}{3}y$

$x - \frac{2}{3}y = \frac{1}{3}$

$-\frac{2}{3} + 10 = \frac{-2}{3} + \frac{30}{3} = \frac{28}{3}$

$\frac{3}{7}(\frac{7}{3}y = \frac{28}{3})$ $\leftarrow \frac{3}{7}R_2$

$\frac{\cancel{28}^{4}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{7}}$

$\downarrow$

$x - \frac{2}{3}y = \frac{1}{3}$ $\leftarrow \frac{2}{3}R_2 + R_1$

$\frac{2}{3}(y = 4)$

$\frac{2}{3} \cdot 4 + \frac{1}{3} = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$

$\downarrow$

$\left.\begin{aligned} x &= 3 \\ y &= 4 \end{aligned}\right\}$ solution.

e.g. $\frac{1}{2}(2x - 6y = -8) \leftarrow \frac{1}{2}R_1$

$-5x + 13y = 1$

$\downarrow$

$5(x - 3y = -4)$

$-5x + 13y = 1 \leftarrow 5R_1 + R_2$

$\downarrow$

$x - 3y = -4$

$-\frac{1}{2}(-2y = -19) \leftarrow -\frac{1}{2}R_2$

$\downarrow$

$x - 3y = -4 \leftarrow \underline{\underline{3}}R_2 + R_1$

$3(y = \frac{19}{2})$

$\downarrow$

$\left.\begin{array}{l} x = \frac{49}{2} \\ y = \frac{19}{2} \end{array}\right\}$ <u>Solution</u>

$3 \cdot \frac{19}{2} - 4 = \frac{57}{2} - \frac{8}{2} = \frac{49}{2}$

e.g. $-\frac{1}{2}(x - 6y = 4)$

$\frac{1}{2}x + 2y = 1 \leftarrow -\frac{1}{2}R_1 + R_2$

$\downarrow$

$-\frac{1}{2}(-6y) + 2y = 3y + 2y = 5y$

$x - 6y = 4$

$\frac{1}{5}(5y = -1) \leftarrow \frac{1}{5}R_2$

$\downarrow$

$x - 6y = 4 \leftarrow \underline{\underline{6}}R_2 + R_1$

$6(y = -\frac{1}{5})$

$\downarrow$

$-\frac{6}{5} + 4 = -\frac{6}{5} + \frac{20}{5} = \frac{14}{5}$

$$\left.\begin{aligned} x &= \frac{14}{5} \\ y &= -\frac{1}{5} \end{aligned}\right\} \text{solution}$$

called "diagonal form"

e.g. $-1(x + 2y + 2z = 11)$

$x - y - z = -4 \leftarrow (-1)R_1 + R_2$

$2x + 5y + 9z = 39$

$\downarrow$

$-2R_1 + R_2$

$-2x - 4y - 4z = -22$

$x - y - z = -4$

$-x - 5y - 5z = -26$

$-2(x + 2y + 2z = 11)$

$-3y - 3z = -15$

$2x + 5y + 9z = 39 \leftarrow -2R_1 + R_3$

$\downarrow$

$x + 2y + 2z = 11$

$-\frac{1}{3}(-3y - 3z = -15) \leftarrow -\frac{1}{3}R_2$

$y + 5z = 17$

$\downarrow$

$x + 2y + 2z = 11$

$-1(y + z = 5)$

$y + 5z = 17 \leftarrow (-1)R_2 + R_3$

$\downarrow$

$x + 2y + 2z = 11 \leftarrow -2R_2 + R_1$

$-2(y + z = 5)$

$4z = 12$

$\downarrow$ 4

$$
\begin{aligned}
x \quad\quad &= 1 \\
y + z &= 5 \\
\tfrac{1}{4}(4z &= 12) \leftarrow \tfrac{1}{4}R_3
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
x \quad\quad &= 1 \\
y + z &= 5 \leftarrow (-1)R_3 + R_2 \\
-1(z &= 3)
\end{aligned}
$$

$$\Downarrow$$

$$
\left.
\begin{aligned}
x \quad\quad &= 1 \\
y \quad &= 2 \\
z &= 3
\end{aligned}
\right\} \text{solution}
$$

(Diagonal Form).