# MATH 110 - EXAM 2 - SOLUTIONS

1. $Pr(E) = .5$ $Pr(F) = .6$ $Pr(E \cup F) = .8$

(a) $Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$

$.8 = .5 + .6 - Pr(E \cap F)$

$\therefore Pr(E \cap F) = .3$ //

(b) $Pr(E|F) = \dfrac{Pr(E \cap F)}{Pr(F)} = \dfrac{.3}{.6} = \dfrac{1}{2}$

(c) $Pr(F|E) = \dfrac{Pr(E \cap F)}{Pr(E)} = \dfrac{.3}{.5} = \dfrac{3}{5}$

(d) Yes because $Pr(E \cap F) = Pr(E) \cdot Pr(F)$

ie. $.3 = (.5)(.6)$

or $Pr(E|F) = Pr(E)$ and $Pr(F|E) = Pr(F)$.

2. (a) $Pr(\text{all red}) = \dfrac{\binom{10}{3}}{\binom{20}{3}} = \dfrac{10 \cdot 9 \cdot 8}{20 \cdot 19 \cdot 18} = \dfrac{2}{19} \approx .105$

(b) # ways to choose 3 different color M+M's

$= 10 \cdot 5 \cdot 5$

# ways to choose red M+M

# ways to choose blue M+M

# ways to choose brown M+M

$$\therefore \Pr\left(\begin{matrix}\text{all different}\\ \text{colors}\end{matrix}\right) = \frac{10 \cdot 5 \cdot 5}{\binom{20}{3}} = \frac{250}{\frac{20 \cdot 19 \cdot 18}{3 \cdot 2}}$$

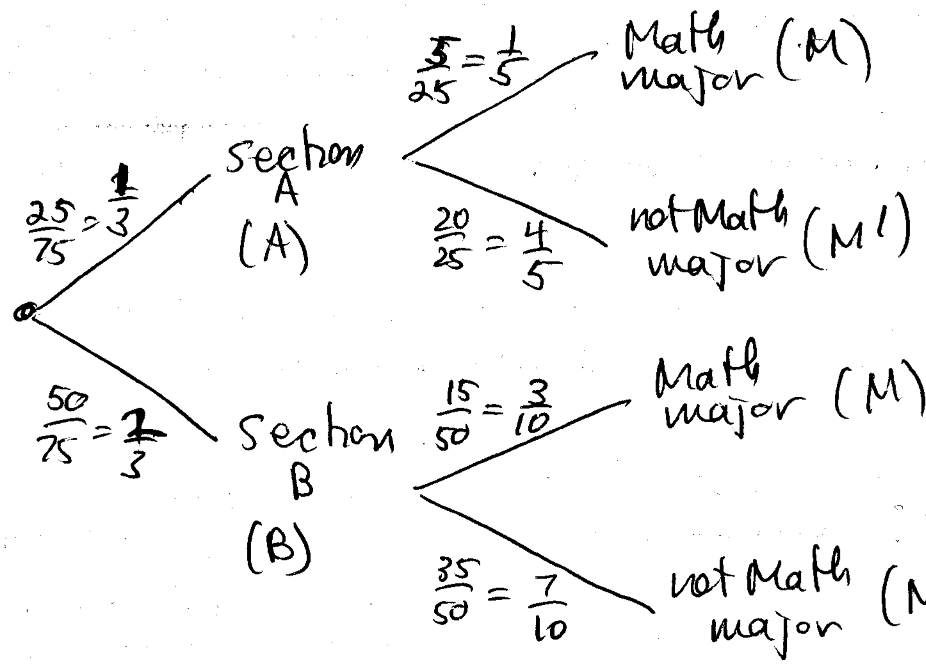
$$= \frac{250}{1140} = \frac{25}{114} \approx .022$$

3. $\Pr\left(\begin{matrix}\text{at least}\\ \text{one win}\end{matrix}\right) = 1 - \Pr\left(\begin{matrix}\text{no}\\ \text{wins}\end{matrix}\right)$

$$\Pr\left(\begin{matrix}\text{no}\\ \text{wins}\end{matrix}\right) = (.99)^{90} \approx .405$$

$$\therefore \Pr\left(\begin{matrix}\text{at least}\\ \text{one win}\end{matrix}\right) \approx 1 - .405 = .595$$

4. (a)



(b) $Pr(M') = Pr(A \cap M') + Pr(B \cap M')$

$= Pr(A)\,Pr(M'|A) + Pr(B)\,Pr(M'|B)$

$= \frac{1}{3} \cdot \frac{4}{5} + \frac{2}{3} \cdot \frac{7}{10}$

$= \frac{4}{15} + \frac{7}{15} = \frac{11}{15}$ //

~~(c)~~ $\cancel{Pr(M|A) = \frac{Pr(M \cap A)}{Pr(A)}}$

(c) $Pr(A|M) = \frac{Pr(A \cap M)}{Pr(M)} = \frac{Pr(A)\,Pr(M|A)}{Pr(M)}$

$= \frac{\frac{1}{3} \cdot \frac{1}{5}}{1 - \frac{11}{15}} = \frac{\frac{1}{15}}{\frac{4}{15}} = \frac{1}{4}$ //

Note: $Pr(M) = 1 - Pr(M') = 1 - \frac{11}{15} = \frac{4}{15}$

from (b).

(d) $Pr(B|M') = \frac{Pr(B \cap M')}{Pr(M')} = \frac{Pr(B)\,Pr(M'|B)}{Pr(M')}$

$= \frac{\frac{2}{3} \cdot \frac{7}{10}}{\frac{11}{15}} = \frac{\frac{7}{15}}{\frac{11}{15}} = \frac{7}{11}$ //