Quiz 13 Section 5.1

Antiderivatives

$$\int f(x)\,dx = F(x) + C$$

$$F'(x) = f(x)$$

$$\int t^{1/2}\,dt = \frac{2}{3}t^{3/2} + C$$

$$\int (e^{2x} + 1)\,dx = \int e^{2x}\,dx + \int 1\,dx$$
$$= \frac{1}{2}e^{2x} + x + C$$

<u>5.2</u> Integration by substitution

$$\int 3x(x^2+1)^{1/2}\,dx$$

Try: $(x^2+1)^{3/2}$. works!

$$\frac{d}{dx}\left((x^2+1)^{3/2}\right) = \frac{3}{\cancel{2}}(x^2+1)^{1/2}(\cancel{2}x)$$
$$= 3x(x^2+1)^{1/2}$$

$$= (x^2+1)^{3/2} + C$$

How do you do this in general?

Idea: How do you tell if the integrand ($f(x)$) comes from an application of chain rule?

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$\int f'(g(x))\, g'(x)\, dx = \underline{f(g(x)) + C}.$$

Key: Find something in the integrand to call $g(x)$.

$$\int f'(g(x))\, \underbrace{g'(x)\, dx}_{du} = \int f'(u)\, du = f(u) + C$$

$$= f(g(x)) + C$$

$u = g(x)$

$\frac{du}{dx} = g'(x) \rightarrow du = g'(x)\,dx$

Hopefully <u>this is easier</u>

<u>e.g.</u> $\int (2x+7)^{1/2}\, dx = \int u^{1/2} \cdot \frac{1}{2}\, du$

$u = 2x+7$

$\frac{du}{dx} = 2 \rightarrow du = 2\,dx$

$dx = \frac{1}{2}\, du$

$$= \frac{1}{2} \int u^{1/2}\, du$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{3/2} + C$$

$$= \frac{1}{3} u^{3/2} + C$$

$$= \frac{1}{3}(2x+7)^{3/2} + C$$

$\underline{\text{e.g.}}$ $\int 8x(4x^2-3)^5\,dx$ $\qquad$ $u = 8x(4x^2-3)$

$u(4x^2-3)^4$

$u = 4x^2-3$

$\frac{du}{dx} = 8x \rightarrow du = 8x\,dx$

$= \int u^5\,du = \frac{1}{6}u^6 + C = \frac{1}{6}(4x^2-3)^6 + C$

$\underline{\text{e.g}}$ $\int \underline{3x}\,\underline{(x^2+1)}^{1/2}\,\underline{dx}$

$3x(x^2+1)^{1/2}$

$((3x)^2(x^2+1))^{1/2}$

$= \underline{\underline{(9x^4+9x^2)^{1/2}}}$

$\underline{\text{NOT}}$ $\text{S}\underline{\text{IMPLE}}\text{R}$

$u = x^2+1$

$\frac{3}{2}du = (2x\,dx)\frac{3}{2}$

$\frac{3}{2}du = \underline{3x\,dx}$

$= \int u^{1/2}\cdot\frac{3}{2}du = \frac{3}{2}\int u^{1/2}du = \frac{3}{2}\frac{2}{3}u^{3/2} + C$

$= u^{3/2} + C = (x^2+1)^{3/2} + C$

$$\int x^2 e^{-x^3}\,dx$$

$u = e^{-x^3}$

$du = -3x^2 e^{-x^3}\,dx$

$-\frac{1}{3}\,du = x^2 e^{-x^3}\,dx$

$$= -\frac{1}{3}\int du = -\frac{1}{3}\int 1\,du$$

$$= -\frac{1}{3}u + c$$

IT WORKED!

$$= -\frac{1}{3}e^{-x^3} + c$$

$$\int x^2 e^{-x^3}\,dx$$

$u = -x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}\,du = x^2\,dx$

$$= \int e^u \cdot -\frac{1}{3}\,du$$

$$= -\frac{1}{3}\int e^u\,du$$

$$= -\frac{1}{3}e^u + c = -\frac{1}{3}e^{-x^3} + c$$

eg $\int \frac{1}{x(\ln(x))^2}\,dx$

$u = x(\ln(x))^2$

$du = \left(x \cdot 2\ln(x)\left(\frac{1}{x}\right) + \ln(x)^2\right) dx$

Does not seem to help.

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{u^2}\,du$

etc...

e.g. $\int e^{-x}(1+e^{2x})\,dx$

$u = e^{2x}$ $\quad u^{1/2} = e^{x}$

$du = 2e^{2x}\,dx$ $\quad u^{-1/2} = e^{-x}$

$= 2u\,dx$

$dx = \frac{du}{2u}$

$= \int u^{-1/2}(1+u)\frac{du}{2u}$

actually works.

OR $\int (e^{-x} + e^{x})\,dx$

etc...