#12 p164

(a) Estimate additional revenue

$R(q) = 240q - .05q^2$

q changes from 80 to 81 new q old q

Δq = change in q = $81 - 80 = 1$

Want to estimate ΔR: change in R.

$$\boxed{\Delta R \cong R'(q)\Delta q} \qquad \frac{\Delta R}{\Delta q} \approx R'(q)$$

$R'(q) = 240 - .10q$

$\Delta R \cong R'(80)\Delta q = (240 - (.10)(80))(1)$

$= 232$ dollars

(b) Actual (or exact) additional revenue.

$\Delta R = R(81) - R(80)$

$= (240(81) - .05(81)^2) - (240(80) - .05(80)^2)$

$= 19111.95 - 18880$

$= 231.95$ dollars

Common mistake:

For (b) $R'(81) - R'(80)$

$= (240 - .10(81)) - (240 - .10(80))$

$= -8.1 + 8.0 = -.1 \leftarrow \frac{\text{dollars}}{\text{unit}}$

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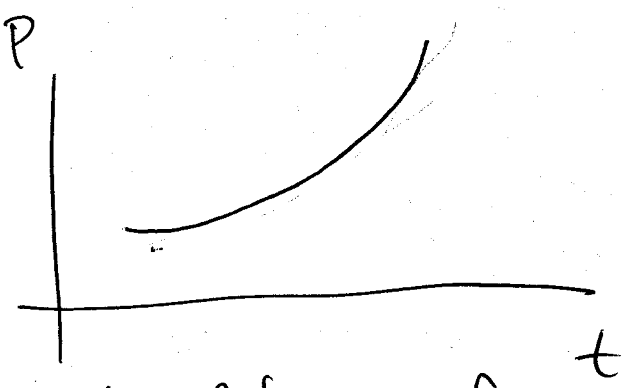
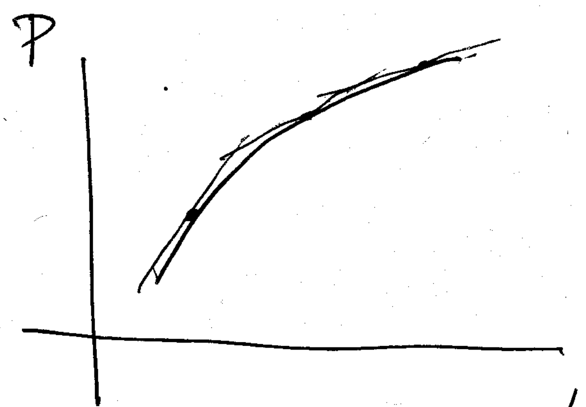
22) $P(t) = -t^3 + 9t^2 + 48t + 200$

(a) $R(t) = P'(t) = -3t^2 + 18t + 48 \leftarrow \frac{\text{thousands}}{\text{year}}$

(b) Rate of change of growth rate.



Rate of change of growth rate is positive



Rate of change of growth rate is negative

$R'(t) = P''(t) = -6t + 18 \leftarrow \frac{\text{thousands}/\text{year}}{\text{year}}$

(c) $\Delta R \approx R'(t)\Delta t$

$\Delta t = 4\frac{1}{12} - 4 = \frac{1}{12}$

$\Delta R \approx R'(4)\left(\frac{1}{12}\right) = (-6 \cdot 4 + 18)\left(\frac{1}{12}\right) = (-6)\left(\frac{1}{12}\right) = -.5$ $\frac{\text{thousands}}{\text{year}}$

2.6. Implicit Differentiation and ~~Related~~ Rates

Example

Find $\dfrac{dy}{dx}$ if $x + \dfrac{1}{y} = 4.$

$$x + \frac{1}{y} = 4$$

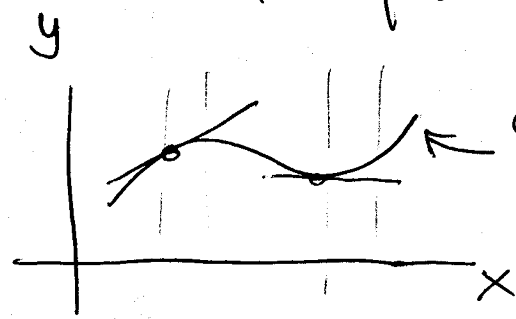
$$\frac{1}{y} = 4 - x$$

$$y = \frac{1}{4-x} = (4-x)^{-1}$$

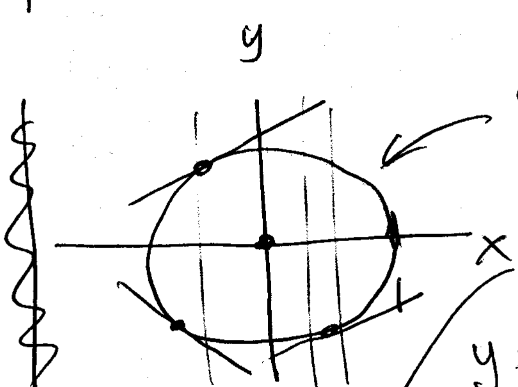
$$\frac{dy}{dx} = (-1)(4-x)^{-2}(-1)$$

$$= (4-x)^{-2} = \frac{1}{(4-x)^2}$$

Idea: Curve in ~~space~~ the plane



graph of $y = f(x)$



graph of $x^2+y^2=1$

$y = g(x)$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

Implicit Differentiation

Suppose an equation defines y *implicitly* as a differentiable function of x. To find the derivative of y,

1. Differentiate both sides of the equation with respect to x. Remember that y is really a *function of* x and use the chain rule when differentiating terms containing y.
2. Solve the differentiated equation algebraically for $\frac{dy}{dx}$.

Example

Find $\frac{dy}{dx}$ using implicit differentiation if $x + \frac{1}{y} = 4$.

Assume y is a function of x (and not solve for y in terms of x)

$$x + \frac{1}{y(x)} = 4$$

$$\frac{d}{dx}\left(x + \frac{1}{y(x)}\right) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}\left(\frac{1}{y(x)}\right) = \frac{d}{dx}\left(y(x)^{-1}\right) = (-1)(y(x))^{-2}\left(\frac{dy}{dx}\right)$$

$$1 - \frac{1}{y(x)^2}\frac{dy}{dx} = 0$$

$$1 - \frac{1}{y^2}\frac{dy}{dx} = 0 \longrightarrow 1 = \frac{1}{y^2}\frac{dy}{dx}$$

$$\frac{dy}{dx} = y^2$$

Since $y = \frac{1}{4-x}$

$$y^2 = \frac{1}{(4-x)^2}$$