Quiz 8 3.3, 3.4 (Wednesday)
Exam 2 Monday March 28 2.5 – 3.5
calculator is permitted

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Section 3.5
What do we know?
- $f'$ tells us where $f$ is increasing/decreasing
  ($f' > 0 \Rightarrow f$ increasing, $f' < 0 \Rightarrow f$ decreasing)
- If $f'(c) = 0$ or $f'(c)$ does not exist, $c$ is a critical point. Local extrema always occur at critical points.
- $f''$ tells us concavity of $f$
  ($f'' > 0 \Rightarrow$ concave up, $f'' < 0 \Rightarrow$ concave down)

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p270 #4) "$xy^2$ is as large as possible"

Let $A = xy^2$. Want to maximize $A$.

~~Cont~~ Constraint: $x \geq 0$, $y \geq 0$, $x + y = 30$

Write $A$ as a function of one variable.

$x + y = 30 \Rightarrow y = 30 - x$

$A = xy^2 = x(30-x)^2$

Problem now: Find $x$ that maximizes $A = x(30-x)^2$

Find critical numbers of A:

$$A' = x(2(30-x)(-1)) + (30-x)^2(1)$$

$$\cancel{= (30-x)(-2x+1)} \qquad \cancel{x=30}$$

$$\cancel{x=\tfrac{1}{2}}$$

$$\cancel{(30-x)(-2x+1)=0}$$

$$\cancel{A(30)=0} \qquad \cancel{A(\tfrac{1}{2}) = \tfrac{1}{2}(30-\tfrac{1}{2})^2 = \tfrac{1}{2}}$$

$$= (30-x)(-2x+30-x)$$

$$= (30-x)(-3x+30)$$

$$(30-x)(-3x+30) = 0 \qquad x=30$$

$$x=10$$

$$A(30) = 0$$

$$A(10) = 10(30-10)^2 = 4000$$

$$x+y=30$$

$$10+y=30$$

$$A(0) = 0$$

$$\downarrow$$

$A = xy^2$ is maximized when $x=10,\ y=20$

#10) $x$ – price of each flashlight

$P$ – profit generated at price $x$.

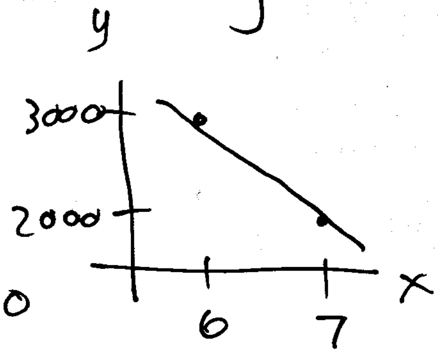
Want to find $x$ that maximizes $P$.

1. Find an expression for $P$ in terms of $x$.

Revenue: $R = x\underbrace{(\#\text{ flashlights sold at price } x)}_{y}$

$x=6 \quad y=3000$

$x=7 \quad y=2000$




$$m = \frac{3000-2000}{6-7} = -1000$$

$$y - 3000 = -1000(x-6)$$

$$y = -1000x + 6000 + 3000$$

$$= -1000x + 9000.$$

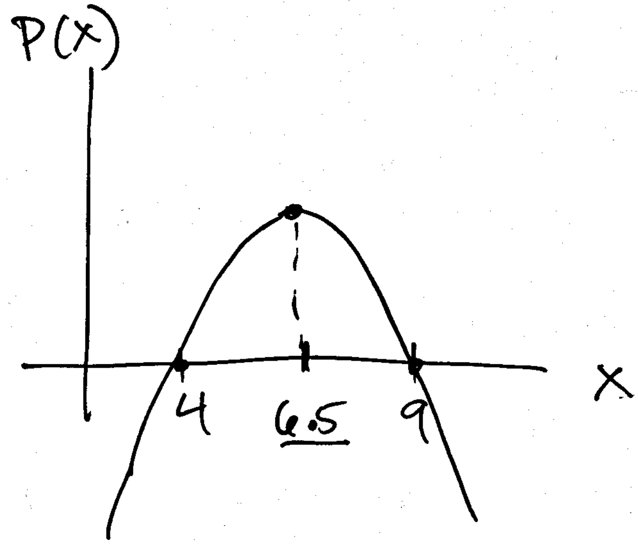
$$R(x) = x(-1000x+9000) = 1000x(-x+9)$$

Cost: $C = 4(\#\text{ flashlights sold at price } x) = 4y$

$$C(x) = 4(-1000x+9000) = 4000(-x+9)$$

$$P(x) = R(x) - C(x) = 1000x(9-x) - 4000(9-x)$$

$$= (9-x)(1000x-4000) = 1000(9-x)(x-4).$$




$P(x) = 1000(9-x)(x-4)$

$P'(x) = 1000[(9-x)(1) + (x-4)(-1)]$

$= 1000(9-x-x+4)$

$= 1000(-2x+13)$

$1000(-2x+13) = 0$

$\underline{x = 6.5}$

Price of \$6.50 generates maximum profit.

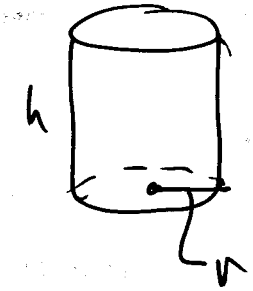
Max profit is $P(6.5) = 1000(9-6.5)(6.5-4)$

$= 1000(2.5)^2$

$= 1000(6.25)$

$=$ \$6250 (per month)

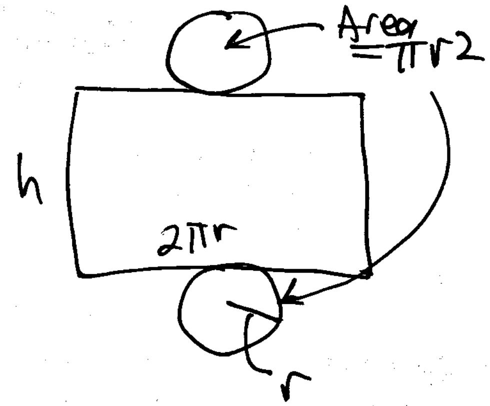
#25) Let $S$ = surface area of can




Find $h, r$ that minimize $S$.

$$S = 2\pi r h + 2\pi r^2$$

Constraint:




$6.89\pi$ = volume of can

$= \pi r^2 h$

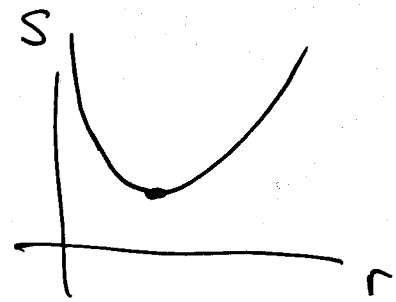
$$r^2 h = 6.89$$

$$h = \frac{6.89}{r^2}$$

$$S = 2\pi \cancel{r}\left(\frac{6.89}{r^{\cancel{2}}}\right) + 2\pi r^2 = 2\pi \frac{6.89}{r} + 2\pi r^2$$

$$= 2\pi\left(\frac{6.89}{r} + r^2\right) = 2\pi\left(6.89r^{-1} + r^2\right)$$

$$S' = 2\pi\left(-6.89r^{-2} + 2r\right)$$

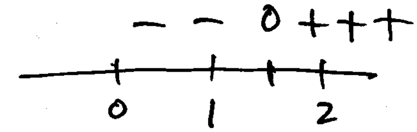



$$-6.89r^{-2} + 2r = 0$$

$$2r = 6.89r^{-2}$$

$$r^3 = \frac{6.89}{2} = 3.445$$

$$r = (3.445)^{1/3} \approx 1.51 \text{ in.}$$

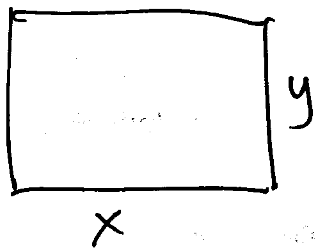



$$S'(1) = 2\pi(-6.89 + 2) < 0$$

$$S'(2) = 2\pi\left(-\frac{6.89}{4} + 4\right) > 0$$

$$h \approx \frac{6.89}{(1.51)^2} \approx 3.02 \text{ in.}$$

#12) $A$ = area of field



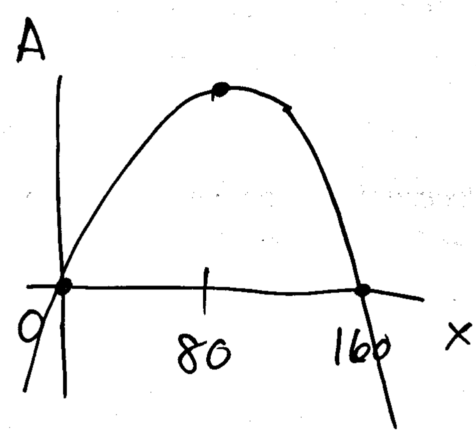

$$A = xy$$

Constraint: $2x + 2y = 320$

$$x + y = 160$$

$$y = 160 - x$$

$$A = x(160 - x)$$




Area maximized when $\underline{x = 80}$ and

$y = 160 - 80 = \underline{80 \text{ yards}}$