Quiz 12 4.4 (Calculators allowed)

Exam 3 Monday 3.4, 3.5, 4.1–4.4.

3.4 7, ~~8~~

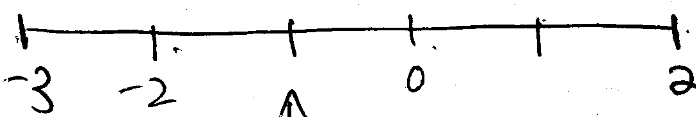
3.5 23

4.1 27

4.4 19, 5

3.4 7) $f(x) = (x^2-4)^5 \quad -3 \le x \le 2$

Idea:



find all critical numbers in here.

Check f at critical numbers and at $x=-3$, $x=2$.

$$f'(x) = 5(x^2-4)^4(2x)$$
$$= 10x(x^2-4)^4$$
$$10x(x^2-4)^4 = 0$$
$$10x(x+2)^4(x-2)^4 = 0 \qquad x=0, x=-2, x=2$$

$f(-3) = ((-3)^2 - 4)^5 = 5^5 = 3125$ //

$f(-2) = ((-2)^2 - 4)^5 = 0$

$f(0) = (-4)^5 = -1024$ //

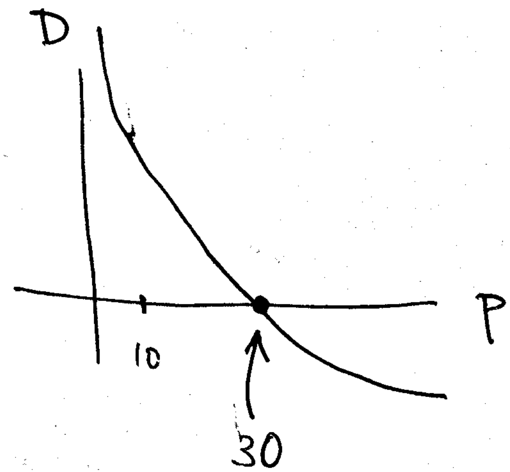
$f(2) = (2^2 - 4)^5 = 0$

Abs minimum of $f(x)$ on $[-3,2]$ is -1024 at $x=0$.

Abs. max of $f(x)$ on $[-3,2]$ is 3125 at $x=-3$

3.4 27) $D(p) = \frac{3000}{p} - 100$; $p = 10$

p – price D – # of units of some commodity



$$E(p) = \frac{p}{q}\frac{dq}{dp} \qquad q = D(p), \quad \frac{dq}{dp} = D'(p)$$

$$= \frac{p}{D(p)} \cdot D'(p)$$

$$E(p) = \frac{\cancel{p}}{\frac{3000}{p} - 100} \cdot \frac{-3000}{p^{\cancel{2}}}$$

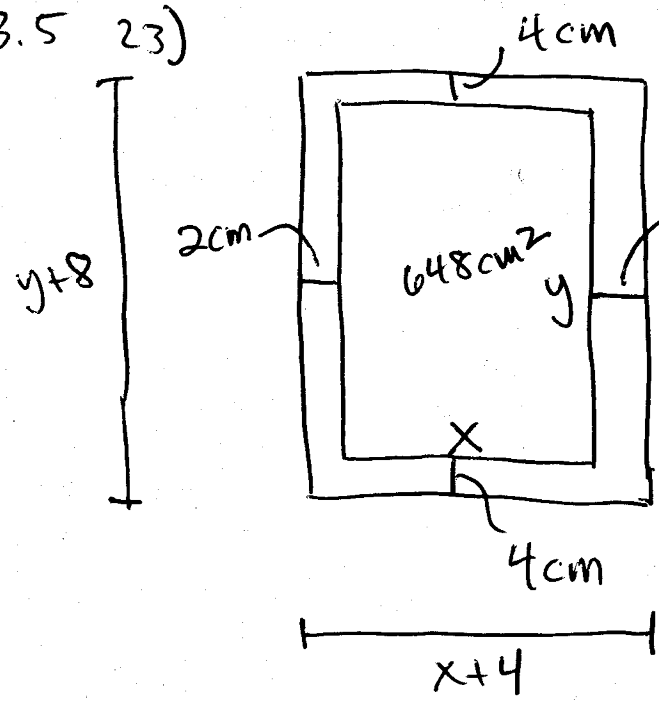
$$= \frac{-3000}{3000 - 100p} \; //$$

$$D'(p) = \frac{-3000}{p^2}$$

Elasticity at $p=10$: $|E(10)| = \left| \frac{-3000}{3000 - 100(10)} \right| = \frac{3}{2}$

elastic.

3.5 23)



Want to minimize area of the paper.

$$A = (x+4)(y+8)$$

constraint:

$$648 = xy$$

$$y = \frac{648}{x}$$

$$A = (x+4)\left(\frac{648}{x}+8\right)$$

$$A' = (x+4)\left(\frac{-648}{x^2}\right) + \left(\frac{648}{x}+8\right)(1)$$

$$= \cancel{\frac{-648}{x}} - \frac{4(648)}{x^2} + \cancel{\frac{648}{x}} + 8$$

$$= \frac{-4(648)}{x^2} + 8$$

$$\boxed{\text{Dimensions minimizing area are } x=18,\ y=36}$$

$$\frac{-4(648)}{x^2} + 8 = 0$$

$$8 = \frac{4(648)}{x^2}$$

$$8x^2 = 4(648)$$

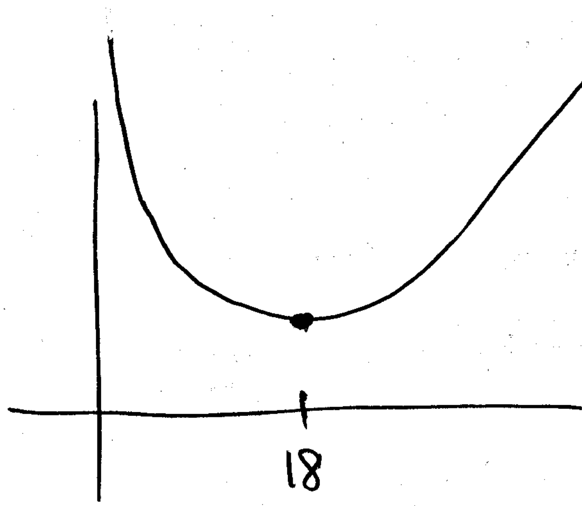
$$x^2 = \frac{\cancel{4}(648)}{\cancel{8}2} = 324 \longrightarrow x = 18 \quad \cancel{x = -18}$$

$$y = \frac{648}{18} = 36$$

What are possible values for x?

$x > 0 \leftarrow$ this is only limitation

Look at $A = (x+4)\left(\frac{648}{x} + 8\right)$



OR Find A''. $A' = -\frac{4(648)}{x^2} + 8 = -4(648)x^{-2} + 8$

$A'' = 8(648)x^{-3}$

$A''(18) = 8(648)\frac{1}{(18)^3} > 0$

$\therefore$ A is concave up at $x = 18$

$\therefore$ $x = 18$ is a minimum.

4.1 27) $\left(\frac{1}{8}\right)^{x-1} = 2^{3-2x^2}$ $\qquad \frac{1}{8} = 2^{-3}$

$$\left(2^{-3}\right)^{x-1} = 2^{3-2x^2}$$

$$2^{-3x+3} = 2^{3-2x^2}$$

$$-3x+\cancel{3} = \cancel{3}-2x^2$$

$$-3x = -2x^2$$

$$2x^2-3x = 0$$

$$x(2x-3) = 0$$

$$x = 0 \quad x = \frac{3}{2}.$$

With logarithms

$$\left(\frac{1}{8}\right)^{x-1} = 2^{3-2x^2}$$

$$\ln\left(\left(\frac{1}{8}\right)^{x-1}\right) = \ln\left(2^{3-2x^2}\right)$$

$$(x-1)\underbrace{\ln\left(\frac{1}{8}\right)}_{\ln(2^{-3})} = (3-2x^2)\ln(2)$$

$$(x-1)(-3)\cancel{\ln(2)} = (3-2x^2)\cancel{\ln(2)}$$

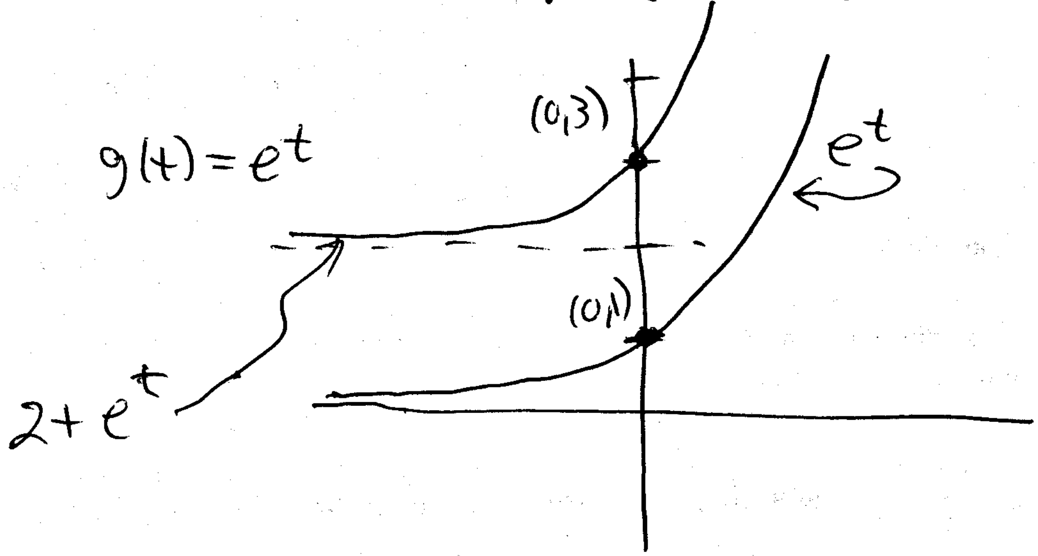
$$-3x+3 = 3-2x^2 \quad \text{etc...}$$

4.4 5) $f(t) = 2 + e^t$

incr/decr. $f'(t) = e^t$ $\qquad$ $f'(t) > 0$ all t

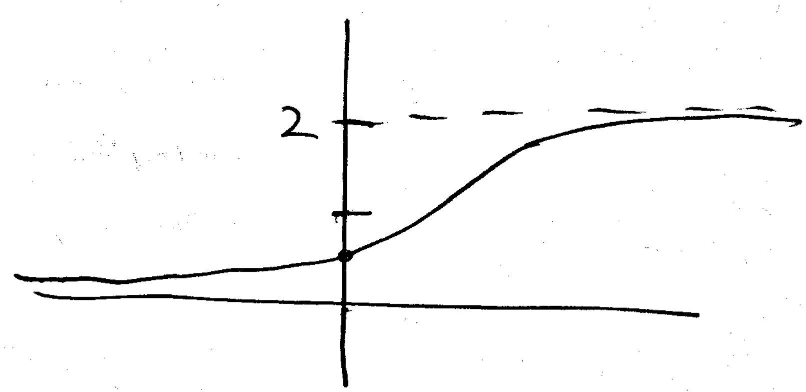
$e^t = 0$ no soln. $\qquad$ so f is increasing

no crit #s. $\qquad$ everywhere.



9) $f(x) = \dfrac{2}{1+3e^{-2x}}$

$f(0) = \dfrac{2}{1+3} = \dfrac{1}{2}$



e^{-2x}