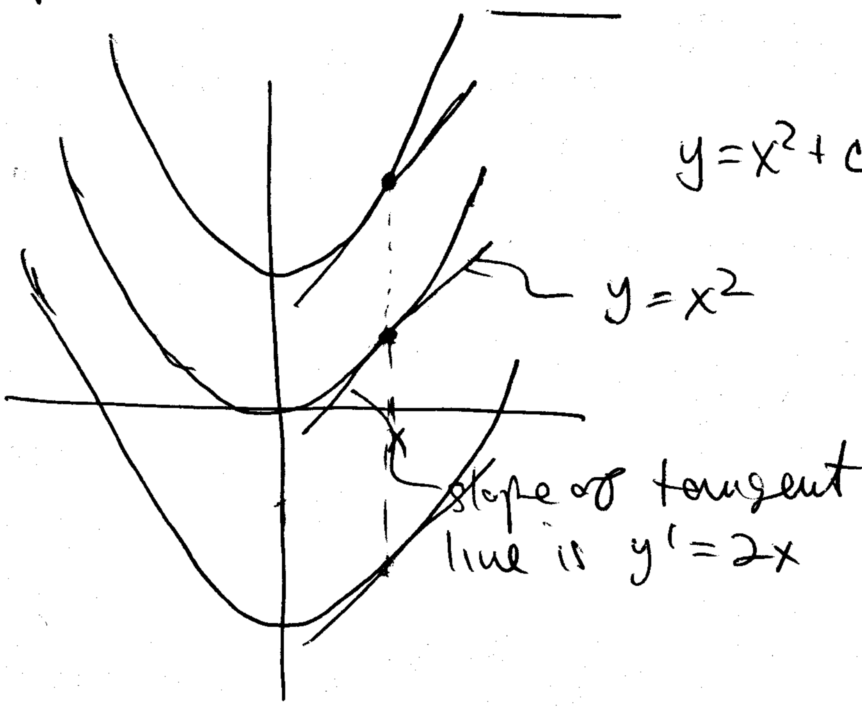
Quiz 12 Wednesday 4.4, 5.1

Exam 3 Monday 4/25 3.4, 3.5, 4.1 – 4.4.

$\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$

Interpretation of "$\underline{\underline{+C}}$":

~~$y=(x-c)^2$~~



The Indefinite Integral

Fundamental property of Antiderivatives

If $F(x)$ is an antiderivative of the continuous function $f(x)$, then any other antiderivative of $f(x)$ has the form $F(x) + C$ for some constant C.

The Indefinite Integral

The family of all antiderivatives of $f(x)$ is written

$$\int f(x)\, dx = F(x) + C$$

and is called the indefinite integral of $f(x)$.

The integral symbol is $\int$, the function $f(x)$ is called the integrand, C is the constant of integration, and dx is a differential that indicates x is the variable of integration.

$\int \leftarrow$ remind you of an "S"

Problem: Find $\int f(x)\,dx$

Idea: Recognize $f(x)$ as the derivative of something.

eg $\int 3x^2\,dx = x^3 + C$

We know $\frac{d}{dx}(x^3) = \underline{3x^2}$

$\int 6x^2\,dx = 2x^3 + C$

$6x^2 = 2 \cdot \underline{3x^2}$ $\quad \cancel{\frac{d}{dx}(2\cdot 3x^2) = 2\frac{d}{dx}(}$

$\frac{d}{dx}(2\cdot x^3) = 2\frac{d}{dx}(x^3)$

$= 2\cdot 3x^2 = 6x^2$

$\int x^4\,dx =$

$\frac{1}{5}x^5 + C$

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\quad x^4$

want $nx^{n-1} = x^4$, $n=5$

$\frac{d}{dx}(x^5) = 5x^4$

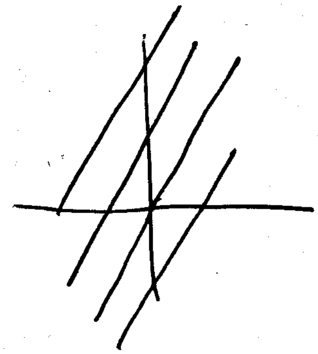
$\frac{d}{dx}\left(\frac{1}{5}x^5\right) = \frac{1}{5}\frac{d}{dx}(x^5) = \frac{1}{5}\cdot 5x^4$

$= x^4$

$$\int \frac{1}{x}\,dx = \ln(x) + c.$$

$$\int 3\,dx = 3x + c$$

$$\int k\,dx = kx + c.$$



$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c$$

if $n \neq -1$

$$\frac{d}{dx}\left(x^{n+1}\right) = (n+1)x^n$$

$$\frac{d}{dx}\left(\frac{1}{n+1}x^{n+1}\right) = \frac{1}{\cancel{n+1}}\cancel{(n+1)}x^n$$

$$= x^n$$

$$\int \frac{1}{x}\,dx = \ln(x) + c.$$

$$\int e^x\,dx = e^x + c$$

$$\frac{d}{dx}\left(e^{3x}\right) = 3e^{3x}$$

$$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + c$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + c.$$

Rules for Integrating Common Functions

- The constant rule: $\int k \, dx = kx + C$ for constant k
- The power rule: $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$ for all $n \neq -1$
- The logarithmic rule: $\int \dfrac{1}{x} \, dx = \ln |x| + C$ for all $x \neq 0$
- The exponential rule: $\int e^{kx} \, dx = \dfrac{1}{k} e^{kx} + C$ for $k \neq 0$

Example

Find these integrals:

a. $\int x^{15} \, dx \quad = \dfrac{1}{16} x^{16} + C$

b. $\int e^{3x} \, dx \quad = \frac{1}{3} e^{3x} + C.$

Algebraic Rules for Indefinite Integration

- The constant multiple rule:

$$\int k f(x)\ dx = k \int f(x)\ dx \quad \text{for constant } k$$

- The sum rule:

$$\int [f(x) + g(x)]\ dx = \int f(x)\ dx + \int g(x)\ dx$$

- The difference rule:

$$\int [f(x) - g(x)]\ dx = \int f(x)\ dx - \int g(x)\ dx$$

NOT TRUE $\int f(x)g(x)\,dx \neq \int f(x)dx \cdot \int g(x)dx$

$$\int \frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

The Indefinite Integral

Example (#6)

Find the indefinite integral

$$\int 3e^x \, dx$$

$$= 3\int e^x \, dx$$

$$= 3e^x + C$$

$$\frac{d}{dx}(3e^x) = 3\,\frac{d}{dx}(e^x) = 3e^x$$

$$\int e^{3x} \, dx = \frac{1}{3}e^{3x} + C$$

The Indefinite Integral

Example (#10)

Find the indefinite integral

$$\int \left(\frac{1}{x^2} - \frac{1}{x^3} \right) \, dx$$

$$= \int \frac{1}{x^2} dx - \int \frac{1}{x^3} dx$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$= \int x^{-2} dx - \int x^{-3} dx$$

$$= \frac{1}{-1} x^{-1} - \frac{1}{-2} x^{-2} + C$$

$$= -x^{-1} + \frac{1}{2} x^{-2} + C$$

$$= \frac{-1}{x} + \frac{1}{2x^2} + C.$$

The Indefinite Integral

Example (#22)

Find the indefinite integral

$$\int y^3 \left(2y + \frac{1}{y}\right)\, dy$$

$$= \int (2y^4 + y^2)\, dy$$

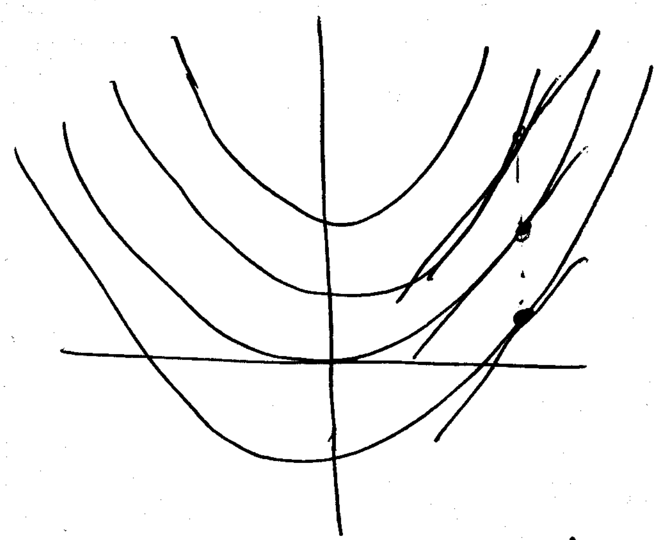
$$= 2\int y^4\, dy + \int y^2\, dy$$

$$= 2\frac{1}{5}y^5 + \frac{1}{3}y^3 + c$$

$$= \frac{2}{5}y^5 + \frac{1}{3}y^3 + C.$$

Interpreting the "+ C"

(1) Add constant to $F(x)$
does not change the slope.



(2) The +C is an "initial value"

For example:

$s(t)$ = position at time t.

(think of $s(t)$ as the "odometer reading")

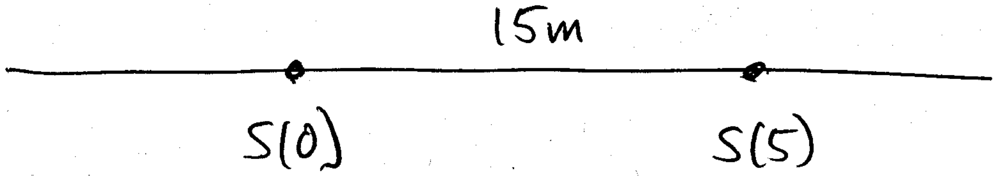
$s'(t)$ = rate of change of position
= velocity
(think of speedometer reading).

Say $s'(t) = 3 \frac{m}{sec.}$ for all t

(constant speed)

Can you find your position at $t = 5$ sec.

i.e. can you find $s(5)$? 15 m



$s'(t)$ can only tell me my relative position not my position.

To find $s(t)$ I need to know where I started from, or my position at any instant of time.

In this example: $s'(t) = 3$

$$s(t) = 3t \boxed{+c}$$

$\boxed{\text{What is } c?}$ $s(0) = 3 \cdot 0 + c = c$

$$s(t) = 3t + \underline{s(0)}.$$

OR $s(t) = 3t + c$. I know $s(1)$.

$s(1) = 3 \cdot 1 + c = c + 3$ $c = s(1) - 3$ so $s(t) = 3t + s(1) - 3$

The Initial Value Problem

A differential equation is an equation that involves derivatives. An initial value problem is a problem that involves solving a differential equation subject to a specified initial condition.

Example (#34)

Solve the initial value problem:

$$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}} \quad \text{where} \quad y = 5 \quad \text{when} \quad x = 4$$

→ this means find the function $y = f(x)$ that satisfies $\frac{dy}{dx} = f'(x) = \frac{x+1}{\sqrt{x}}$ and

$5 = f(4)$

$$f(x) = \int \frac{x+1}{\sqrt{x}}\,dx = \int \frac{x+1}{x^{1/2}}\,dx$$

$$= \int \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}\,dx = \int (x^{1/2} + x^{-1/2})\,dx$$

$$= \frac{2}{3}x^{3/2} + 2x^{1/2} + c$$

what is c?

$f(4) = 5$

$f(4) = \frac{2}{3}(4)^{3/2} + 2(4)^{1/2} + c$

$= \frac{2}{3}(8) + 2 \cdot 2 + c = \frac{16}{3} + 4 + c$

$= \frac{28}{3} + c$

$\frac{28}{3} + c = 5$

$c = 5 - \frac{28}{3} = \frac{15}{3} - \frac{28}{3} = -\frac{13}{3}$ //

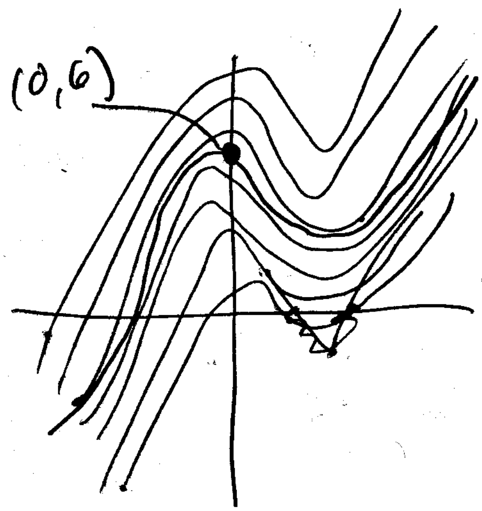
$\therefore f(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} - \frac{13}{3}$ //

The Initial Value Problem

Example (#36)

Find the function $f(x)$ whose tangent line has slope $3x^2 + 6x - 2$ for each value of x and whose graph passes through the point $(0, 6)$.

$$f'(x) = 3x^2 + 6x - 2$$



$$f(x) = \int (3x^2 + 6x - 2)\,dx = x^3 + 3x^2 - 2x + C$$

Find C.

$$\boxed{f(0) = 6}$$

$$f(0) = (0)^3 + 3(0)^2 - 2(0) + C = C \qquad \therefore\ C = 6$$

$$f(x) = x^3 + 3x^2 - 2x + 6$$