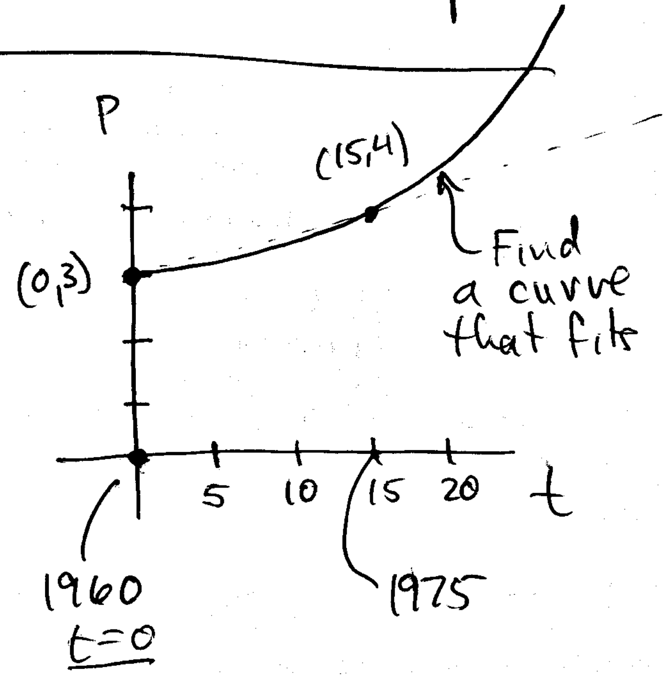
Quiz 11 4.2, 4.3

Exam 3 Monday 4/25 3.4, 3.5, 4.1–4.4
Calculators are permitted.

#24)



1960	3 billion
1975	4 billion

t = # years after 1960

P = population in billions

$$\boxed{P(t) = 3e^{kt}}$$

1. Find k
2. Solve $P(t) = 40$

1. We know $P(15) = 4$ Also know $P(15) = 3e^{15k}$

$$3e^{15k} = 4$$
$$e^{15k} = \frac{4}{3}$$
$$\ln(e^{15k}) = \ln\left(\frac{4}{3}\right)$$
$$15k = \ln(4/3)$$
$$k = \frac{\ln(4/3)}{15} \approx .019$$

So $P(t) = 3e^{.019t}$

2. Solve $P(t) = 40$

$$3e^{.019t} = 40$$

$$e^{.019t} = \frac{40}{3}$$

$$\ln(e^{.019t}) = \ln\left(\frac{40}{3}\right)$$

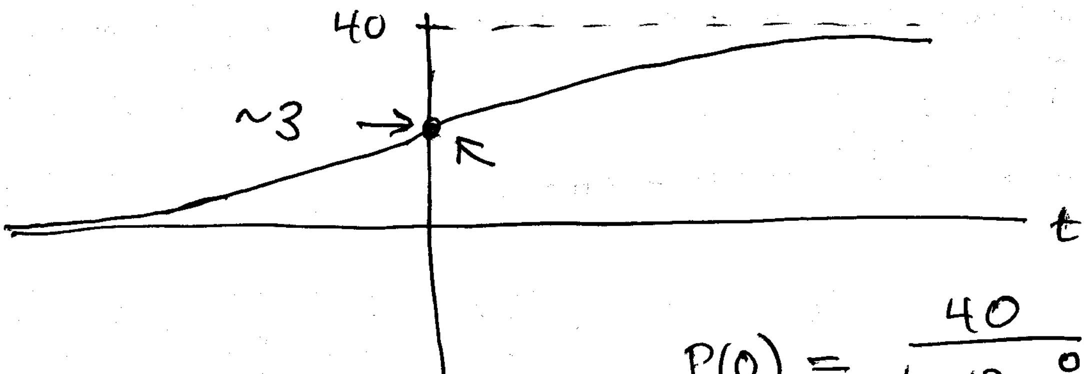
$$.019t = \ln\left(\frac{40}{3}\right)$$

$$t = \frac{\ln(40/3)}{.019} \approx 135 \text{ years}$$

$$\begin{array}{r} 1960 \\ 135 \\ \hline 2095 \end{array}$$

Pop will hit 40 billion in 2095.

#38) $P(t) = \dfrac{40}{1+12e^{-.08t}}$ Logistic model



$$P(0) = \frac{40}{1+12e^{0}} = \frac{40}{13} \approx 3.1$$

$$\lim_{t \to +\infty} \frac{40}{1+12e^{-.08t}} = 40$$

$$\lim_{t \to -\infty} \frac{40}{1+12e^{-.08t}} = 0$$

(a) Find rate of increase of P in 2000 $(t=40)$

$$P'(t) = \frac{-40\left(12e^{-.08t}(-.08)\right)}{(1+12e^{-.08t})^2}$$

$$= \frac{38.4\,e^{-.08t}}{(1+12e^{-.08t})^2}$$

$$P'(40) = \frac{38.4\,e^{-.08(40)}}{(1+12e^{-.08(40)})^2} \approx .71 \ \frac{\text{billions}}{\text{year}}$$

so 710 million people/year.

(b) When will pop grow most rapidly?
Find maximum value of $P'(t)$.

$$P''(t) = \frac{(1+12e^{-.08t})^{\cancel{2}}(38.4(-.08)e^{-.08t}) - (38.4e^{-.08t})\left(2\cancel{(1+12e^{-.08t})}(12(-.08)e^{-.08t})\right)}{(1+12e^{-.08t})^4}$$

$$= \frac{\cancel{(1+12e^{-.08t})}(38.4e^{-.08t})\left[-.08(1+12e^{-.08t}) - 24(-.08)e^{-.08t}\right]}{(1+12e^{-.08t})^{\cancel{4}\,3}}$$

$$= \frac{(38.4e^{-.08t})\left(-.08\left[1+12e^{-.08t} - 24e^{-.08t}\right]\right)}{(\quad\text{——}\quad)^3}$$

$$= \frac{(38.4e^{-.08t})(-.08)(1-12e^{-.08t})}{(\quad\text{——}\quad)^3} = 0$$

$$1 - 12e^{-.08t} = 0$$

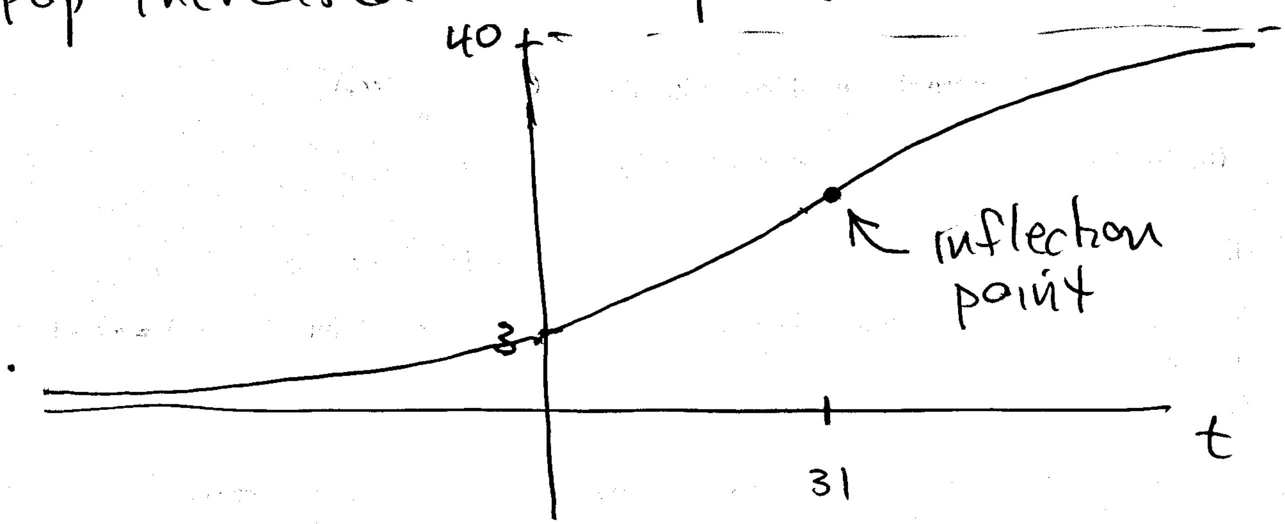
$$1 = 12e^{-.08t}$$

$$e^{-.08t} = \frac{1}{12}$$

$$-.08t = \ln\left(\frac{1}{12}\right)$$

$$t = \frac{\ln\left(\frac{1}{12}\right)}{-.08} \approx 31.0 \longrightarrow \begin{array}{r} 1960 \\ +\ 31 \\ \hline 1991 \end{array}$$

Pop increased most rapidly in 1991



5.1. Antidifferentiation: The Indefinite Integral

Antidifferentiation

A function $F(x)$ is said to be an antiderivative of $f(x)$ if

$$F'(x) = f(x)$$

for every x in the domain of $f(x)$. The process of finding antiderivatives is called antidifferentiation or indefinite integration.

Example

Verify that $F(x) = \frac{1}{3}x^3 + 5x + 2$ is an antiderivative of $f(x) = x^2 + 5$.

e.g. $f(x) = 3x^2 + 5x \xrightarrow{\frac{d}{dx}} f'(x) = 6x + 5$

$? \xleftarrow{\int} g(x) = 6x + 5$

antidifferentiation
(integration)

$F'(x) = x^2 + 5 \qquad F'(x) = f(x)$

$$\frac{1}{3}x^3 \xrightarrow{\frac{d}{dx}} 3 \cdot \frac{1}{3}x^2 = x^2$$

$$G(x) = \underbrace{\frac{1}{3}x^3 + 5x}\ -7$$

$$G'(x) = x^2 + 5 = f(x)$$

$$H(x) = \underbrace{\frac{1}{3}x^3 + 5x}\ + 38$$

$$H'(x) = x^2 + 5 = f(x)$$

Important fact: The full family of antiderivatives to $f(x) = x^2 + 5$

is $F(x) = \underbrace{\frac{1}{3}x^3 + 5x}\ + C$

arbitrary constant.