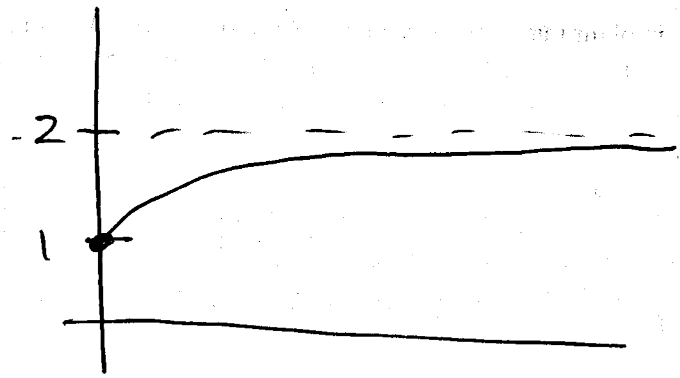
Quiz 11, Sections 4.2, 4.3

4.4 Exponential Models

#2



Know $f(0)=1$ Horizontal asymptote at $y=2$

$f_1(x) = 2 - e^{-2x}$ $(f_1(0)=1)$

$f_4(x) = \dfrac{2}{1+e^{-x}}$ $(f_4(0)=1)$

$\ln(0) = y$

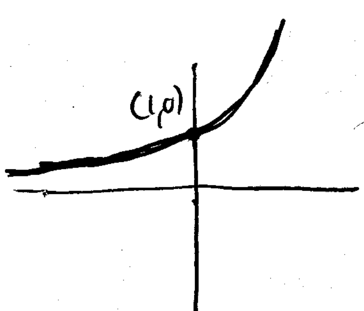
$\log_e(0) = y$

$e^y = 0$

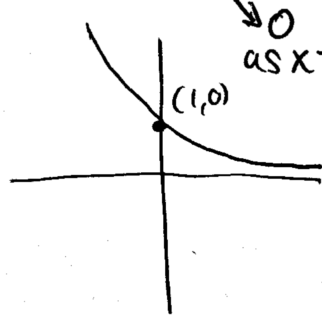
$\ln(0)$ undefined

$\ln(1)=0$

$$\lim_{x\to\infty} f_1(x) = \lim_{x\to\infty} (2-\underbrace{e^{-2x}}_{\to 0 \text{ as } x\to\infty}) = 2$$

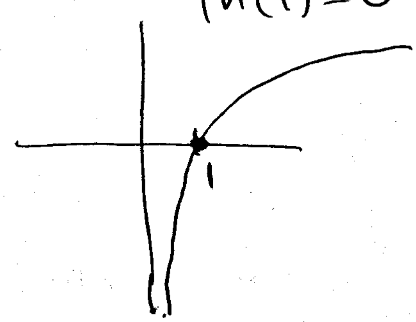


e^x



$e^{-x} = \left(\frac{1}{e}\right)^x$

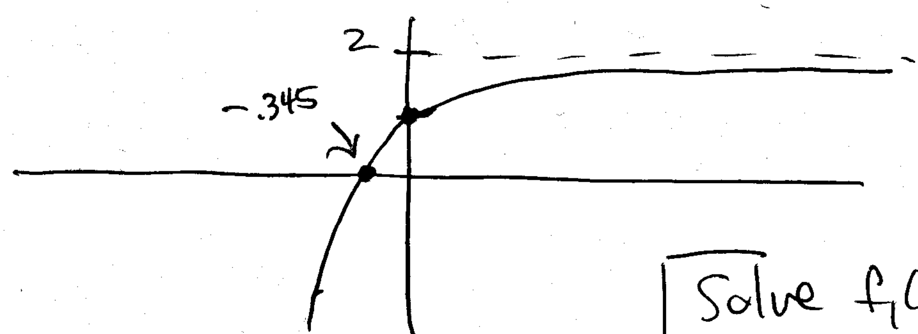
$0<\frac{1}{e}<1$



$$\lim_{x\to\infty} f_4(x) = \lim_{x\to\infty} \frac{2}{1+\underbrace{e^{-x}}_{\substack{\to 0 \\ \text{as } x\to\infty}}} = \frac{2}{1} = 2$$

Both are consistent with the graph.

What do graphs look like for $x<0$?



Look at $f_1(x) = 2 - e^{-2x}$

As $x \to -\infty$ $e^{-2x} \to +\infty$

so $2 - e^{-2x} \to -\infty$

Solve $f_1(x) = 0$.

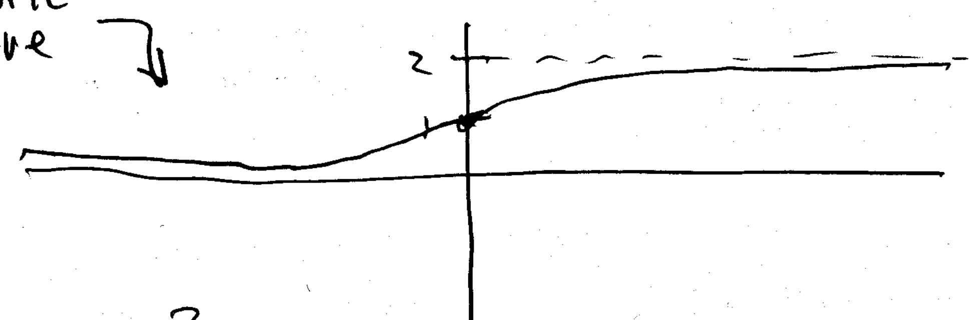
$2 - e^{-2x} = 0$

$2 = e^{-2x}$

$\ln(2) = \ln(e^{-2x}) = -2x$

$x = -\frac{\ln(2)}{2} \approx -.345$

Logistic curve



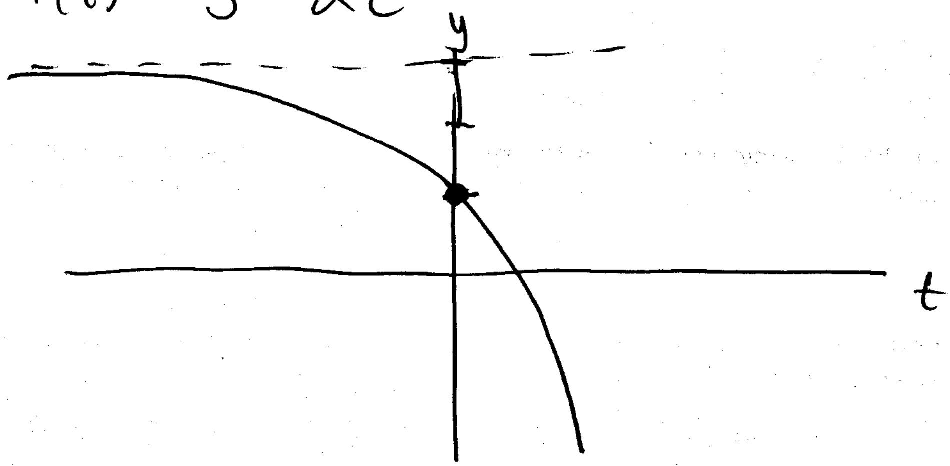
$f_4(x) = \frac{2}{1+e^{-x}}$ As $x \to -\infty$, $e^{-x} \to +\infty$

$1+e^{-x} \to +\infty$ $\frac{2}{1+e^{-x}} \to 0$

$y=0$ is also a horizontal asymptote!

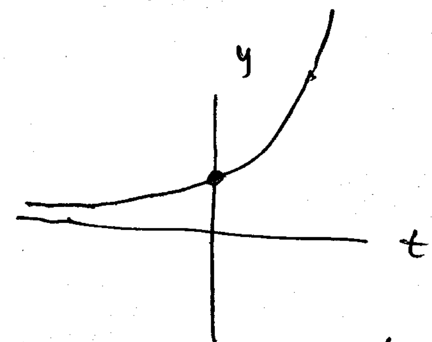
#8) $f(t) = 3 - 2e^t$

$f(0) = 3 - 2e^0 = 3 - 2 = 1$



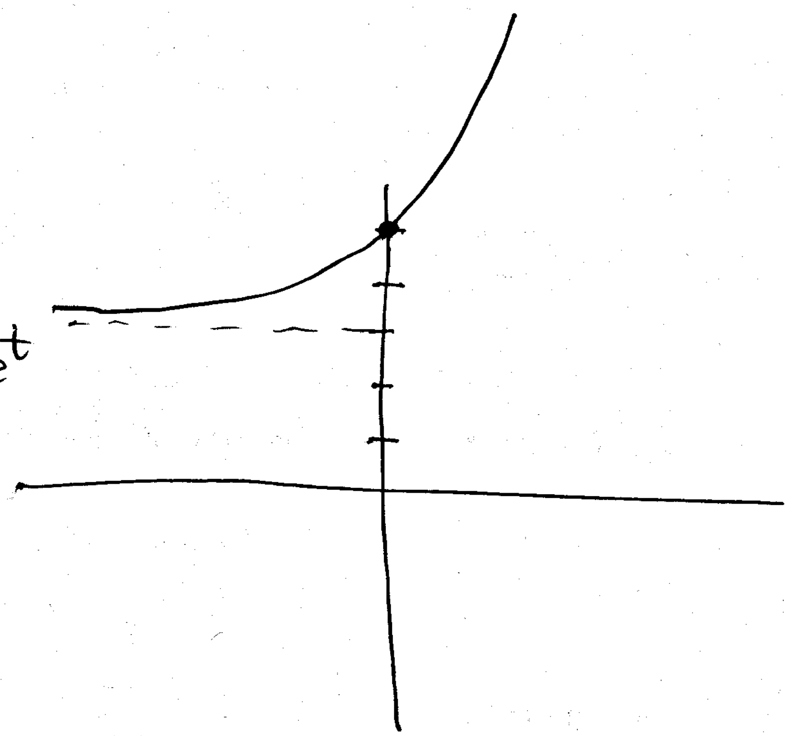
~~Hor~~ Horizontal asymptotes:

$$\lim_{t \to -\infty} 3 - 2\underbrace{e^t}_{\substack{\downarrow \\ 0 \\ \text{as } t \to -\infty}} = 3$$



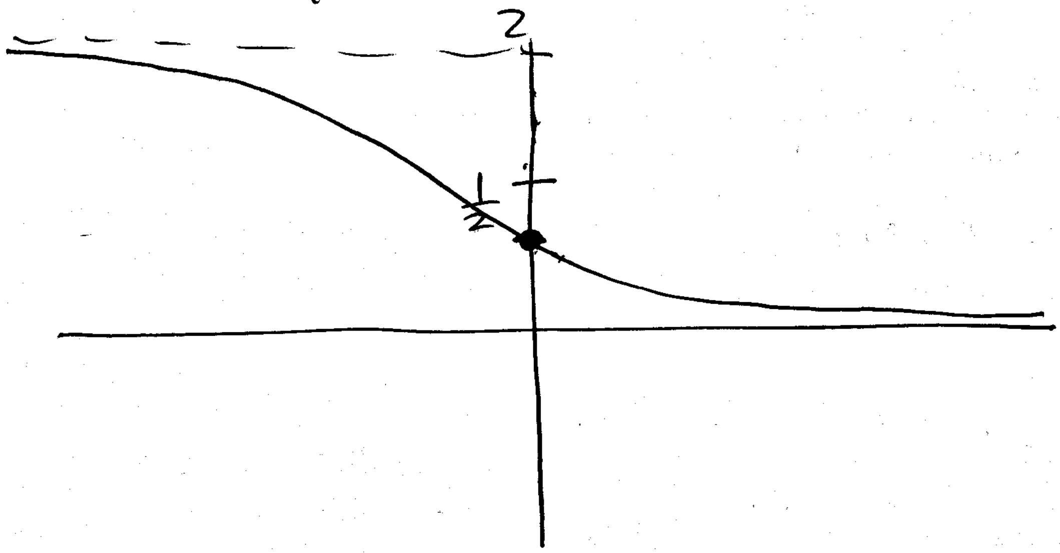
e^t

$$\lim_{t \to \infty} 3 - 2\underbrace{e^t}_{\substack{\downarrow \\ +\infty \\ \text{as } t \to \infty.}} = -\infty$$



Look at $g(t) = 3 + 2e^t$

#10 $h(t) = \dfrac{2}{1+3e^{2t}}$ Logistic



Horizontal Asymptotes:

$$\lim_{t\to\infty} \frac{2}{\underbrace{1+3e^{2t}}_{\substack{\downarrow \\ \infty \\ \text{as } t\to\infty}}} = 0$$

$$\lim_{t\to-\infty} \frac{2}{1+3e^{2t}} = 2$$

y-intercept

$$h(0) = \frac{2}{1+3e^{0}} = \frac{1}{2}$$

#12) $f(x) = xe^{-x}$

$$\underline{\lim_{x \to \infty} xe^{-x} = 0}$$

Find critical #s, etc.

$$f'(x) = x\left(e^{-x}(-1)\right) + e^{-x}(1)$$

$$= -xe^{-x} + e^{-x}$$

$$= e^{-x}(1-x)$$

$e^{-x}(1-x) = 0$ $\quad \underline{\underline{x=1}}$

rel. max.

f': $+ + +$ 0 $- - -$ (at 0, 1, 2)

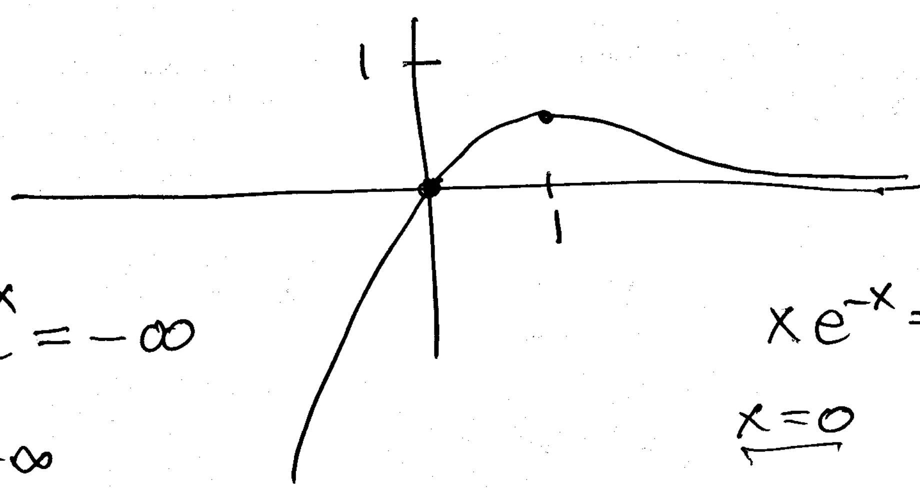
$f'(0) = (1)(1) > 0$

$f'(2) = (+)(-1) < 0$

Asymptotes (horizontal)

$f(0) = 0$

$f(1) = 1 \cdot e^{-1}$



$$\lim_{x \to -\infty} \underbrace{\tilde{x}}_{-\infty} \underbrace{e^{-x}}_{+\infty} = -\infty$$

$xe^{-x} = 0$

$\underline{x = 0}$ $\quad \cancel{e^{-x} = 0}$

#14) $f(x) = e^{-x^2}$

critical #s etc.

$$f'(x) = e^{-x^2}(-2x) = -2xe^{-x^2}$$

$$-2xe^{-x^2} = 0 \longrightarrow -2x = 0 \quad \cancel{e^{-x^2} = 0}$$

$$x = 0$$

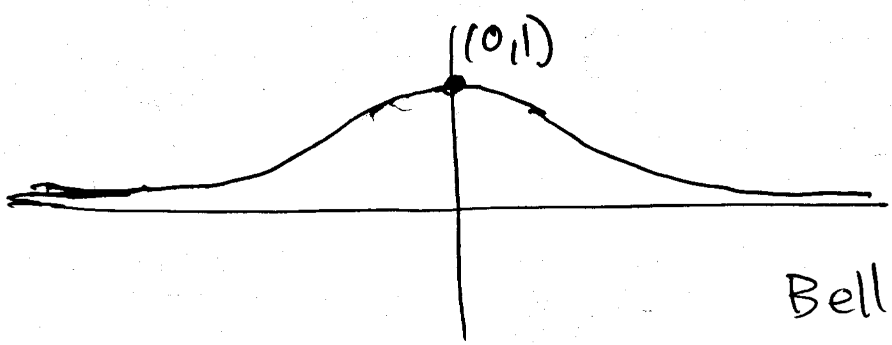
$\underline{x = 0}$

f' + + + 0 − − −

−1 0 1

$f(0) = 1$

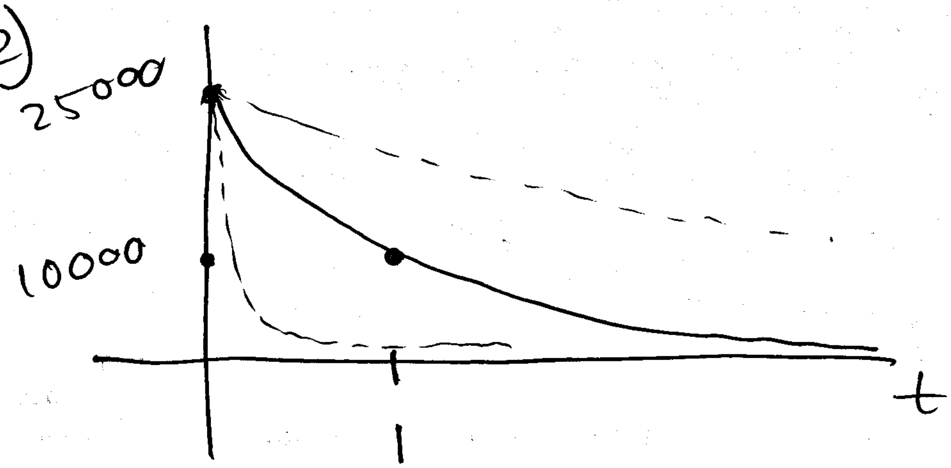
$$f'(-1) = (2)(+) > 0$$
$$f'(1) = (-2)(+) < 0$$



Bell curve

Gaussian curve

#22)



$f(t) =$ sales in #books/month
t months after publicity stops.

$$f(t) = 25000\, e^{-kt}$$

Find k so that $f(1) = 10000$

$$f(1) = 25000\, e^{-k(1)} = 25000\, e^{-k}$$

$$10000 = 25000\, e^{-k} \quad \text{Solve for } k$$

$$\frac{10000}{25000} = e^{-k}$$

$$.4 = e^{-k}$$

$$\ln(.4) = \ln(e^{-k}) = -k$$

$$k = -\ln(.4) \approx .92$$

$$\rightarrow f(t) = 25000\, e^{-.92t}$$

$$f(2) = 25000\, e^{-.92(2)}$$

$$\cong 25000\,(.16)$$

$$= 4000 \text{ books/month}$$

1 month $25000 \rightarrow 10000$ factor of $.4$

2 month $10000 \rightarrow ?$ factor of $.4$

$\underline{4000}$