5.3 31, 37, 9, 21 5.1 15+

5.4 8, 9

5.2 13, 23, 31, 39

4.3 anything. 4.1 33

NO CALCULATORS PERMITTED ON FINAL

5.1 #15) $\int \left(\frac{e^x}{2} + x\sqrt{x}\right) dx$

$= \int \left(\frac{1}{2}e^x + \underbrace{x\sqrt{x}}_{x^1 \cdot x^{1/2} = x^{3/2}}\right) dx$

$= \frac{1}{2}e^x + \int x^{3/2}\,dx$

$= \frac{1}{2}e^x + \frac{2}{5}x^{5/2} + C$

$\int x^r dx = \frac{1}{r+1}x^{r+1} + C$

$\int \frac{1}{x}dx = \ln(x) + C$

$\int e^x dx = e^x + C$

19) $\int \left(\frac{x^2+2x+1}{x^2}\right) dx$

$= \int \left(\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}\right) dx$

$= \int \left(1 + \frac{2}{x} + \frac{1}{x^2}\right) dx = x + 2\ln(x) - x^{-1} + C$

② $\frac{1}{x}$

$\frac{1}{x^2} = x^{-2}$

$\int x^{-2}\,dx = \frac{1}{-1}x^{-1} = -x^{-1}$

$\frac{3}{2} + 1 = \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$

$\frac{1}{\frac{5}{2}}x^{5/2}$

$1 = x^0$ $\frac{1}{1}x^1 = x$

23) $\int \sqrt{t}\,(t^2-1)\,dt = \int t^{1/2}(t^2-1)\,dt$

$= \int t^{5/2} - t^{1/2}\,dt = \frac{2}{7}t^{7/2} - \frac{2}{3}t^{3/2} + C$

25) $\int (e^t+1)^2\,dt = \int (e^{2t} + 2e^t + 1)\,dt$

$= \frac{1}{2}e^{2t} + 2e^t + t + C$

$\frac{1}{2}e^{2t} \xrightarrow{\frac{d}{dt}} 2\,e^{2t}\cdot 2 = 4e^{2t}$

$\int 2e^t\,dt = 2\int e^t\,dt$

$e^{t^2} \xrightarrow{\frac{d}{dt}} 2te^{t^2}$

OR $\int (e^t+1)^2\,dt$

$= \int u^2 \frac{du}{u-1}$

$= \int \frac{u^2}{u-1}\,du.$

$u = e^t + 1 \quad e^t = u-1$

$du = e^t\,dt$

$= (u-1)\,dt$

$dt = \frac{du}{u-1}$

33) $\frac{dy}{dx} = \frac{2}{x} - \frac{1}{x^2}$

$y = -1$ when $x = 1$

$$y = \int \left(\frac{2}{x} - \frac{1}{x^2}\right) dx$$

$$= 2\ln(x) + \frac{1}{x} + C$$

$$-1 = 2\ln(1) + 1 + C$$

($\ln(1) = 0$)

$$-2 = C \quad \checkmark$$

$$y = 2\ln(x) + \frac{1}{x} - 2$$

5.2 13) $\int x^2(x^3+1)^{3/4}\,dx$

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$dx = \frac{du}{3x^2}$

$\int x^2 u^{3/4}\frac{du}{3x^2} = \frac{1}{3}\int u^{3/4}\,du.$

$= \frac{1}{3}\int u^{3/4}\,du.$

$= \frac{1}{3}\cdot\frac{4}{7}u^{7/4}+C$

$= \frac{4}{21}u^{7/4}+C$

$= \frac{4}{21}(x^3+1)^{7/4}+C$

$\int x(x^3+1)^{3/4}\,dx \quad u = x^3+1$

$du = 3x^2\,dx$

$\int \frac{1}{3x}u^{3/4}\,du \quad \frac{1}{3x}du = x\,dx$

$u = x^3+1$

$\frac{1}{3x}\int u^{3/4}\,du \quad x^3 = u-1$

$x = (u-1)^{1/3}$

$\int \frac{1}{3(u-1)^{1/3}}u^{3/4}\,du$

23) $\int \frac{\ln(5x)}{x}\,dx$

$u = \ln(5x)$

$du = \frac{1}{5x} \cdot 5\,dx = \frac{1}{x}\,dx$

$= \int u'\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln(5x))^2 + C.$

$u = \ln(5x) = \ln(5) + \ln(x)$

$du = (0 + \frac{1}{x})\,dx$

31) $\int \frac{x}{2x+1}\,dx$

$u = 2x+1 \quad 2x = u-1$

$x = \frac{1}{2}(u-1)$

$du = 2\,dx$

$\frac{1}{2}du = dx$

~~$du = 3x^2\,dx$~~

~~$\frac{1}{3x^2}du = dx$~~

~~$\frac{1}{3}du = (x^2\,dx)$~~

$= \int \frac{1}{2}(u-1)\frac{1}{u} \cdot \frac{1}{2}\,du$

$= \frac{1}{4}\int \frac{u-1}{u}\,du = \frac{1}{4}\int \frac{u}{u} - \frac{1}{u}\,du = \frac{1}{4}\int (1 - \frac{1}{u})\,du$

$= \frac{1}{4}(u - \ln(u)) + C = \frac{1}{4}(2x+1 - \ln(2x+1)) + C.$

5.3 9) $\int_0^1 e^{-x}(4-e^x)\,dx$

$= \int_0^1 (4e^{-x} - 1)\,dx$

$= (-4e^{-x} - x)\Big|_0^1 = (-4e^{-1} - 1) - (-4 - 0)$

$= -4e^{-1} - 1 + 4 = -4e^{-1} + 3$ //

21) $\int_0^4 \frac{1}{\sqrt{6t+1}}\,dt$

$= \frac{1}{3}(6t+1)^{1/2}\Big|_0^4$

$= \frac{1}{3}(6\cdot 4+1)^{1/2} - \frac{1}{3}(6\cdot 0+1)^{1/2}$

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$ //

$\int \frac{1}{\sqrt{6t+1}}\,dt$ $\quad u = \sqrt{6t+1} = (6t+1)^{1/2}$

$du = \frac{1}{2}(6t+1)^{-1/2}\cdot 6\,dt$

$= 3\frac{1}{\sqrt{6t+1}}\,dt$

$= 3\cdot\frac{1}{u}\,dt$

$\frac{u}{3}\,du = dt$

$= \int \frac{1}{u}\cdot\frac{u}{3}\,du$

$= \int \frac{1}{3}\,du$

$= \frac{1}{3}u = \frac{1}{3}(6t+1)^{1/2}$

$\int \frac{1}{\sqrt{6t+1}}\,dt$ $\quad u = 6t+1$

$du = 6\,dt$

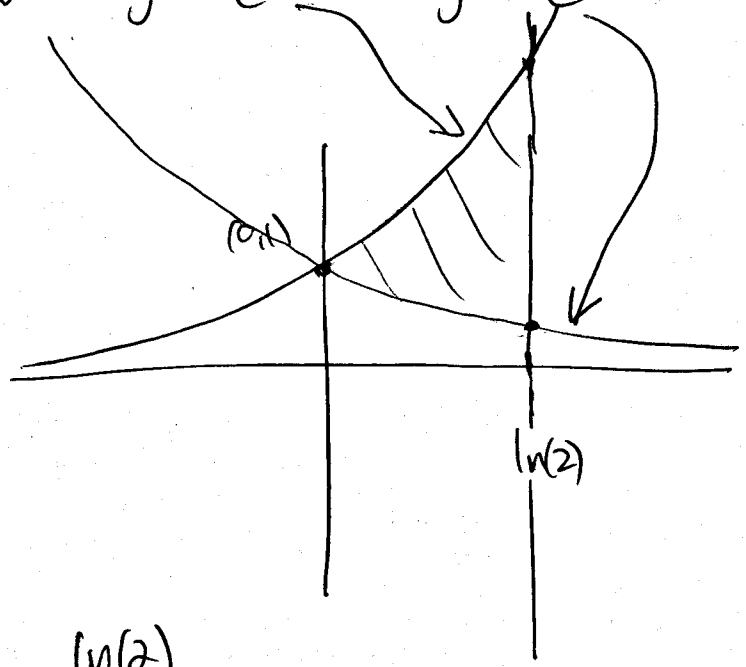
$\frac{1}{6}\,du = dt$

$= \int \frac{1}{\sqrt{u}}\cdot\frac{1}{6}\,du$

$= \frac{1}{6}\int u^{-1/2}\,du = \frac{1}{6}\cdot 2u^{1/2} = \frac{1}{3}u^{1/2} = \frac{1}{3}(6t+1)^{1/2}$

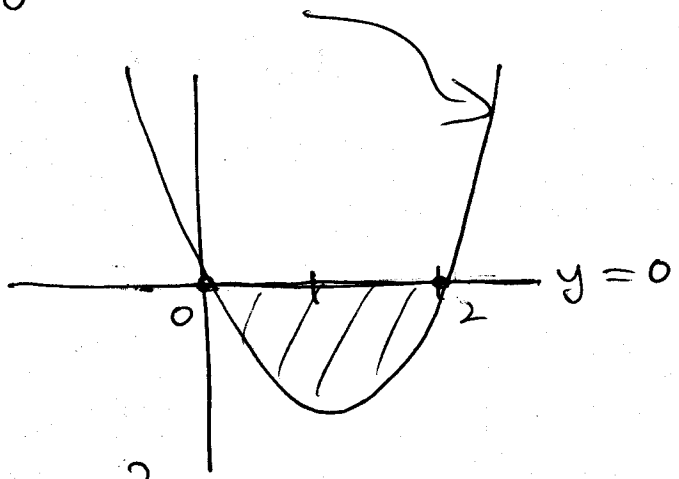
$\frac{32}{7} - \frac{21}{19} + \frac{3}{13}$

5.4 8) $y = e^x$ $y = e^{-x}$ $x = \ln(2)$



$$A = \int_0^{\ln(2)} (e^x - e^{-x})\,dx$$

9) $y = x^2 - 2x$ x-axis.



$$A = \int_0^2 0 - (x^2 - 2x)\,dx = \int_0^2 -x^2 + 2x\,dx$$

$$= -\frac{1}{3}x^3 + x^2 \Big|_0^2 = -\frac{1}{3}(2)^3 + (2)^2 - 0 = -\frac{8}{3} + 4$$

$$= 4/3 \text{ //}$$