MATH 108 – QUIZ 12 – 20 APRIL 2011

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (3 pts. each) Let $f(x) = \dfrac{6}{1+e^{-x}}$.

(a) Find the y-intercept of $f(x)$.

$$f(0) = \frac{6}{1+e^{-(0)}} = \frac{6}{1+1} = 3 \quad y\text{-intercept } (0,3) //$$

(b) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

$$\lim_{x\to\infty} \frac{6}{1+e^{-x}} = \frac{6}{1+0} = 6 //$$

$$\lim_{x\to-\infty} \frac{6}{1+e^{-x}} = 0 //$$

2. (2 pts. each) It is estimated that t years from now, the population of a certain country will be $P(t) = \dfrac{20}{2+3e^{-.06t}}$ million people.

(a) What is the current population?

$$P(0) = \frac{20}{2+3e^{-.06(0)}} = \frac{20}{2+3} = \frac{20}{5} = 4 \text{ million} //$$

(b) What will be the population 30 years from now?

$$P(30) = \frac{20}{2+3e^{-.06(30)}} = \frac{20}{2+3e^{-1.80}} \approx 8.0 \text{ million} //$$

(c) When will be the population reach 6 million?

$$\frac{20}{2+3e^{-.06t}} = 6 \rightarrow 20 = 12 + 18e^{-.06t}$$

$$8 = 18e^{-.06t} \rightarrow e^{-.06t} = \frac{8}{18} = \frac{4}{9}$$

$$\ln e^{-.06t} = \ln\left(\frac{4}{9}\right) \rightarrow -.06t = \ln\left(\frac{4}{9}\right) \rightarrow t = \frac{\ln(4/9)}{-.06}$$

$$\approx 13.5 \text{ yrs} //$$

VI

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (3 pts. each) Let $h(t) = \dfrac{2}{1+3e^{2t}}$.

(a) Find the y-intercept of $h(t)$.

$$h(0) = \frac{2}{1+3e^{2(0)}} = \frac{2}{1+3} = \frac{1}{2}$$

y-intercept $(0, \tfrac{1}{2})$ //

(b) Find $\lim_{t\to\infty} h(t)$ and $\lim_{t\to-\infty} h(t)$.

$$\lim_{t\to\infty} \frac{2}{1+3e^{2t}} = 0 \;//$$

$$\lim_{t\to-\infty} \frac{2}{1+3e^{2t}} = \frac{2}{1+3(0)} = 2 \;//$$

2. (2 pts. each) The temperature of a hot casserole t minutes after it has been removed from the oven is given by $T(t) = 180 - 110\,e^{-.03t}$ degrees Fahrenheit.

(a) What is the temperature of the casserole when it is removed from the oven?

$$T(0) = 180 - 110\,e^{-.03(0)} = 180 - 110 = 70^\circ F$$

(b) What is the temperature of the casserole 30 minutes after it has been removed from the oven?

$$T(30) = 180 - 110\,e^{-.03(30)}$$
$$= 180 - 110\,e^{-.90} \approx 135^\circ F$$

(c) In the long run (that is, as $t \to \infty$), what will be the temperature of the casserole?

$$\lim_{t\to\infty} 180 - 110\,e^{-.03t} = 180^\circ F$$

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