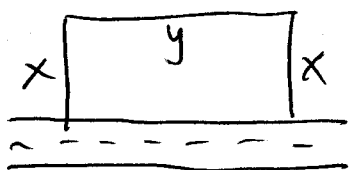Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of $5,000$ square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

Maximize length of fencing

$F = 2x + y \longrightarrow F = 2x + \dfrac{5000}{x}$

$5000 = xy$

$y = \dfrac{5000}{x}$

$F' = 2 - \dfrac{5000}{x^2}$

$2 - \dfrac{5000}{x^2} = 0 \rightarrow 2x^2 = 5000$

$\rightarrow x^2 = 2500 \rightarrow x = \pm 50$

Length of fencing minimized when

$x = 50, \quad y = \dfrac{5000}{50} = 100$

So minimum amount of fencing $F = 2x + y = 200 \text{ yds}$

2. (2 pts. each) Evaluate the following expressions as a simple fraction or whole number.

(a) $16^{3/4} = \left(16^{1/4}\right)^3 = 2^3 = 8 \;//$

(b) $(28 + 36)^{-3/2} = (64)^{-3/2} = \left(64^{1/2}\right)^{-3} = 8^{-3} = \dfrac{1}{512} \;//$

3. (2 pts. each) Use the properties of exponential functions to simplify the given expressions to a single exponential expression.

(a) $\left(\dfrac{x^2}{\sqrt{x}}\right)^{4/3} = \left(\dfrac{x^2}{x^{1/2}}\right)^{4/3} = \left(x^{2-1/2}\right)^{4/3} = \left(x^{3/2}\right)^{4/3} = x^2 \;//$

(b) $(4^{2/3})(2^{4/3}) = \left(2^2\right)^{2/3} \cdot 2^{4/3} = 2^{4/3} \cdot 2^{4/3} = 2^{8/3} \;//$

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular and is to be fenced off on the three sides not adjacent to the highway, and the highway department has 400 feet of fencing available to do the job. What are the dimensions of the picnic area that will enclose the largest area?

Maximize $A = xy$ (area of picnic area)

$400 = 2x + y \longrightarrow y = 400 - 2x$

$A = x(400 - 2x)$

$A' = 400 - 4x$

$400 - 4x = 0$

$x = 100$

$y = 400 - 2(100) = 200$

Largest area when $x = 100$ ft, $y = 200$ ft

2. (2 pts. each) Evaluate the following expressions as a simple fraction or whole number.

(a) $\left(\frac{1}{9}\right)^{3/2} = \left[\left(\frac{1}{9}\right)^{1/2}\right]^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$ //

(b) $(8^{2/3} + 8^{4/3})^{-2} = \left((8^{1/3})^2 + (8^{1/3})^4\right)^{-2} = (2^2 + 2^4)^{-2}$

$= (4 + 16)^{-2} = 20^{-2} = \frac{1}{400}$ //

3. (2 pts. each) Use the properties of exponential functions to simplify the given expressions to a single exponential expression.

(a) $[(y^2)(y^{3/2}]^{4/3} = [y^{2+3/2}]^{4/3} = (y^{7/2})^{4/3} = y^{\frac{7}{2} \cdot \frac{4}{3}} = y^{\frac{14}{3}}$ //

(b) $(25^{2/3})(5^{3/2}) = (5^2)^{2/3} \cdot 5^{3/2} = 5^{4/3} \cdot 5^{3/2}$

$= 5^{4/3 + 3/2} = 5^{17/6}$ //