MATH 108 – QUIZ 9 – 30 MARCH 2011

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) Find the absolute maximum and minimum value of the function $f(x) = 3x^5 - 5x^3 + 1$ on the interval $-1 \leq x \leq 1$.

$f'(x) = 15x^4 - 15x^2$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$x = 0,\ x = 1,\ x = -1$ ← critical numbers

$f(-1) = 3$

$f(0) = 1$

$f(1) = -1$

MAXIMUM VALUE IS 3 AT $x = -1$

MINIMUM VALUE IS -1 AT $x = 1$ //

2. (3 pts. each) Let $p(q)$ be the price in thousands of dollars at which q units of a commodity will be sold, and suppose that $p(q) = 64 - 3q^2$. Let $C(q)$ be the cost in thousands of dollars of producing q units of the commodity, and suppose that $C(q) = q^3 + 16q + 75$. Let $P(q)$ be the profit in thousands of dollars realized by producing q units of the commodity.

(a) Find an expression for the profit function $P(q)$.

$R(q) = q \cdot p(q) = 64q - 3q^3$

$P(q) = R(q) - C(q) = 64q - 3q^3 - q^3 - 16q - 75$

$= -4q^3 + 48q - 75$ //

(b) Find the value of q that maximizes profit.

$P'(q) = -12q^2 + 48$

$-12q^2 + 48 = 0$

$-12(q^2 - 4) = 0$

$q = 2 \quad q = -2$

PROFIT MAXIMIZED AT $q = 2$ UNITS //

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) Find the absolute maximum and minimum value of the function $f(x) = x^3 + 3x^2 + 1$ on the interval $-3 \leq x \leq 2$.

$f'(x) = 3x^2 + 6x$

$3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0 \quad x = -2$ ← critical numbers

$f(-3) = 1$

$f(-2) = 5$

$f(0) = 1$

$f(2) = 21$

MAXIMUM VALUE IS 21 AT $x = 2$

MINIMUM VALUE IS 1 AT $x = -3, x = 0$ //

2. (3 pts. each) Let $p(q)$ be the price in thousands of dollars at which q units of a commodity will be sold, and suppose that $p(q) = 230 - 2q^2$. Let $C(q)$ be the cost in thousands of dollars of producing q units of the commodity, and suppose that $C(q) = q^3 + 5q + 162$. Let $P(q)$ be the profit in thousands of dollars realized by producing q units of the commodity.

(a) Find an expression for the profit function $P(q)$.

$R(q) = q \cdot p(q) = 230q - 2q^3$ REVENUE FUNCTION

$P(q) = R(q) - C(q)$

$= 230q - 2q^3 - q^3 - 5q - 162$

$= -3q^3 + 225q - 162$ //

(b) Find the value of q that maximizes profit.

$P'(q) = -9q^2 + 225$

$-9q^2 + 225 = 0$

$-9(q^2 - 25) = 0$

$q = 5 \quad q = -5$

PROFIT MAXIMIZED AT $q = 5$ UNITS //