MATH 108 – QUIZ 7 – 9 MARCH 2011

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

Let $f(x) = 3x^5 - 5x^3 + 4$.

1. (3 pts. each)

(a) Find the intervals of increase and decrease for the function $f(x)$ given above.

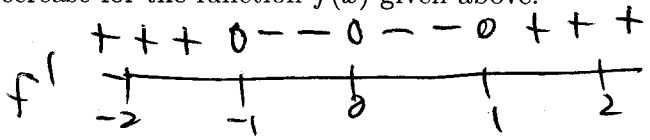
$f'(x) = 15x^4 - 15x^2$

$15x^4 - 15x^2 = 0$

$15x^2(x^2-1) = 0$

$15x^2(x+1)(x-1) = 0$

$x=0 \quad x=-1 \quad x=1$ critical numbers



$f'(-2) > 0 \quad f'(-1/2) < 0 \quad f'(\tfrac{1}{2}) < 0 \quad f'(2) > 0$

increasing $(-\infty, -1) \cup (1, \infty)$

decreasing $(-1, 0) \cup (0, 1)$

(b) Find all critical points (including the y-coordinate) of $f(x)$ and identify them as local maxima, local minima or neither.

$f(0) = 4$ $\quad (0, 4)$ neither

$f(-1) = -3+5+4 = 6$ $\quad (-1, 6)$ local maximum

$f(1) = 3-5+4 = 2$ $\quad (1, 2)$ local minimum

2. (3 pts. each)

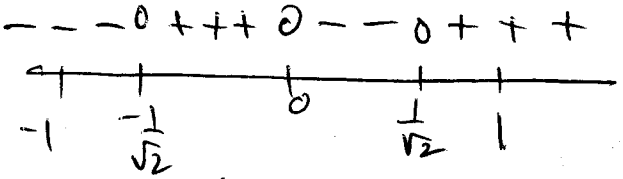
(a) Find the intervals on which the graph of $f(x)$ (given above) is concave up or concave down.

$f''(x) = 60x^3 - 30x$

$60x^3 - 30x = 0$

$30x(2x^2 - 1) = 0$

$x = 0 \quad x = \frac{1}{\sqrt{2}} \quad x = -\frac{1}{\sqrt{2}}$



$f''(-1) < 0 \quad f''(-1/2) > 0 \quad f''(\tfrac{1}{2}) < 0 \quad f''(1) > 0$

concave up $(-\frac{1}{\sqrt{2}}, 0) \cup (\frac{1}{\sqrt{2}}, \infty)$

concave down $(-\infty, -\frac{1}{\sqrt{2}}) \cup (0, \frac{1}{\sqrt{2}})$

(b) Find all inflection points (including the y-coordinate) of $f(x)$.

$f(\frac{1}{\sqrt{2}}) =$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

Let $f(x) = 3x^4 - 8x^3 + 6x^2 + 2$.

1. (3 pts. each)

(a) Find the intervals of increase and decrease for the function $f(x)$ given above.

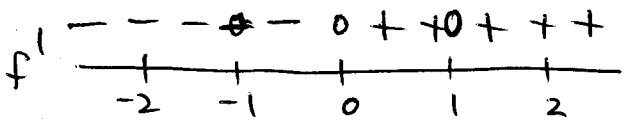
$f'(x) = 12x^3 - 24x^2 + 12x$

$12x^3 - 24x^2 + 12x = 0$

$12x(x^2 - 2x + 1) = 0$

$12x(x-1)^2 = 0$

$x = 0 \quad x = +1$ ← critical numbers



$f'(-1) < 0 \quad f'(+1/2) > 0 \quad f'(2) > 0$

increasing $(0,1) \cup (1,\infty)$

decreasing $(-\infty, 0)$

(b) Find all critical points (including the y-coordinate) of $f(x)$ and identify them as local maxima, local minima or neither.

$f(0) = 2$

$(0,2)$ critical points

$f(1) = 3 - 8 + 6 + 2 = 3 \quad (1,3)$

$(0,2)$ is a local minimum

$(1,3)$ is neither

2. (3 pts. each)

(a) Find the intervals on which the graph of $f(x)$ (given above) is concave up or concave down.

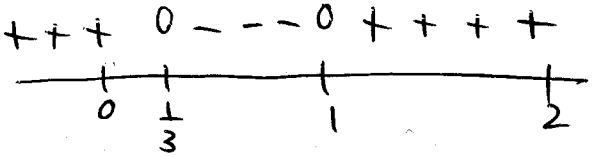
$f''(x) = 36x^2 - 48x + 12$

$36x^2 - 48x + 12 = 0$

$12(3x^2 - 4x + 1) = 0$

$12(3x-1)(x-1) = 0$

$x = 1/3 \quad x = 1$



$f''(0) > 0 \quad f''(\tfrac{1}{2}) < 0 \quad f''(2) > 0$

concave up: $(-\infty, \tfrac{1}{3}) \cup (1, \infty)$

concave down: $(\tfrac{1}{3}, 1)$

(b) Find all inflection points (including the y-coordinate) of $f(x)$.

$f(\tfrac{1}{3}) = \frac{3}{81} - \frac{8}{27} + \frac{6}{9} + 2 = \frac{1}{27} - \frac{8}{27} + \frac{18}{27} + \frac{54}{27} = \frac{65}{27} \quad (\tfrac{1}{3}, \tfrac{65}{27})$

$f(1) = 3 - 8 + 6 + 2 = 3 \quad (1,3)$ ← inflection points