MATH 108 – QUIZ 6 – 2 MARCH 2011

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (a) (4 pts.) Find dy/dx when y is defined implicitly as a function of x by the equation $x^3 - y^2 = 7$.

$$\frac{d}{dx}(x^3 - y^2) = \frac{d}{dx}(7) \qquad \frac{dy}{dx} = \frac{3x^2}{2y} \,//$$

$$3x^2 - 2y\frac{dy}{dx} = 0$$

(b) (2 pts.) Find the slope of the tangent line to the curve defined in part (a) at the point $(2, 1)$.

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \left.\frac{3x^2}{2y}\right|_{(2,1)} = \frac{12}{2} = 6 \,//$$

2. (a) (4 pts.) Find dy/dx by implicit differentiation when y is given as a function of x by the equation $xy + 2y = x^2$.

$$\frac{d}{dx}(xy + 2y) = \frac{d}{dx}(x^2) \qquad x\frac{dy}{dx} + 2\frac{dy}{dx} = 2x - y$$

$$x\frac{dy}{dx} + y + 2\frac{dy}{dx} = 2x \qquad \frac{dy}{dx}(x+2) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x-y}{x+2} \,//$$

(b) (2 pts.) Solving for y in terms of x in the above equation gives $y = \dfrac{x^2}{x+2}$. Find dy/dx from this equation using the quotient rule and show that it is the same as the answer you found in part (a).

$$\frac{d}{dx}\left(\frac{x^2}{x+2}\right) = \frac{(x+2)(2x) - (x^2)(1)}{(x+2)^2} = \frac{2x^2 + 4x - x^2}{(x+2)^2} = \frac{x^2+4x}{(x+2)^2}$$

$$\frac{2x-y}{x+2} = \frac{2x - \frac{x^2}{x+2}}{x+2} = \frac{\frac{2x^2+4x-x^2}{x+2}}{x+2} = \frac{x^2+4x}{(x+2)^2}$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (a) (4 pts.) Find dy/dx when y is defined implicitly as a function of x by the equation $x^2 + y^3 = 12$.

$$\frac{d}{dx}(x^2+y^3) = \frac{d}{dx}(12)$$

$$2x + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{3y^2} //$$

(b) (2 pts.) Find the slope of the tangent line to the curve defined in part (a) at the point $(2, 2)$.

$$\left.\frac{dy}{dx}\right|_{(2,2)} = \left.\frac{-2x}{3y^2}\right|_{(2,2)} = \frac{-4}{12} = -\frac{1}{3} //$$

2. (a) (4 pts.) Find dy/dx by implicit differentiation when y is given as a function of x by the equation $xy + 2y = 3x$.

$$\frac{d}{dx}(xy+2y) = \frac{d}{dx}(3x)$$

$$x\frac{dy}{dx} + y + 2\frac{dy}{dx} = 3$$

$$\to x\frac{dy}{dx} + 2\frac{dy}{dx} = 3 - y$$

$$\frac{dy}{dx}(x+2) = 3 - y$$

$$\frac{dy}{dx} = \frac{3-y}{x+2} //$$

(b) (2 pts.) Solving for y in terms of x in the above equation gives $y = \dfrac{3x}{x+2}$. Find dy/dx from this equation using the quotient rule and show that it is the same as the answer you found in part (a).

$$\frac{d}{dx}\left(\frac{3x}{x+2}\right) = \frac{(x+2)(3)-(3x)(1)}{(x+2)^2} = \frac{6}{(x+2)^2}$$

$$\frac{3-y}{x+2} = \frac{3 - \frac{3x}{x+2}}{x+2} = \frac{\frac{3x+6-3x}{x+2}}{x+2} = \frac{6}{(x+2)^2}$$