

MATH 108 - QUIZ 5 - 23 FEBRUARY 2011

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (3 pts. each) Let $f(x) = x^2 - 3x$.

(a) Use increments to estimate by how much the function $f(x)$ will change when x increases from 5 to 5.5.

$$f'(x) = 2x - 3 \quad \Delta f \approx f'(x) \Delta x \quad \Delta x = 5.5 - 5 = .5$$

$$\Delta f \approx f'(5)(.5) = (2(5) - 3)(.5) = 7(.5) = 3.5 //$$

(b) Find the *exact* change in the function $f(x)$ as x increases from 5 to 5.5.

$$\Delta f = f(5.5) - f(5) = ((5.5)^2 - 3(5.5)) - (5^2 - 3(5))$$

$$= 30.25 - 16.5 - 10$$

$$= 3.75 //$$

2. (3 pts. each) Suppose the total cost in dollars of manufacturing q units of a certain product is $C(q) = 3q^2 + q + 500$.

(a) Use marginal analysis to estimate the cost of manufacturing the 11th unit.

$$\Delta C \approx C'(q) \Delta q \quad \Delta q = 1, \quad C'(q) = 6q + 1$$

$$\Delta C \approx C'(10)(1) = 61 \text{ dollars.} //$$

(b) Find the actual cost of manufacturing the 11th unit.

$$\Delta C = C(11) - C(10) = (3(11)^2 + 11 + 500) - (3(10)^2 + 10 + 500)$$

$$= 374 - 310 = 64 \text{ dollars.} //$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (3 pts. each) Let $f(x) = x^2 + 2x$.

(a) Use increments to estimate by how much the function $f(x)$ will change when x decreases from 4 to 3.5.

$$\Delta f \approx f'(x) \Delta x \quad \Delta x = 3.5 - 4 = -.5$$

$$f'(x) = 2x + 2$$

$$\Delta f \approx f'(4)(-.5) = (2(4) + 2)(-.5) = 10(-.5) = -5 //$$

(b) Find the *exact* change in the function $f(x)$ as x decreases from 4 to 3.5.

$$\begin{aligned} \Delta f &= f(3.5) - f(4) \\ &= [(3.5)^2 + 2(3.5)] - [4^2 + 2(4)] \\ &= 19.25 - 24 = -4.75 // \end{aligned}$$

2. (3 pts. each) A manufacturer's total monthly revenue in dollars generated by producing q units of a certain product is $R(q) = 300q - 3q^2$.

(a) Use marginal analysis to estimate the additional revenue the manufacturer will receive by producing the 21st unit.

$$\Delta R \approx R'(q) \Delta q \quad \Delta q = 21 - 20 = 1$$

$$q = 20 \quad R'(q) = 300 - 6q$$

$$\Delta R \approx R'(20)(1) = 300 - 6(20) = 180 \text{ dollars} //$$

(b) Find the actual additional revenue generated by producing the 21st unit.

$$\begin{aligned} \Delta R &= R(21) - R(20) \\ &= (300(21) - 3(21)^2) - (300(20) - 3(20)^2) \\ &= 4977 - 4800 = 177 \text{ dollars} // \end{aligned}$$