MATH 108 – 25 APRIL 2011– EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given. This exam is counted out of a total of 100 points.

1. (10 pts.) A cylindrical metal can is to have a surface area of 24π square inches. Find the dimensions (height and radius) of such a can that has maximum volume. (Hint: The volume V of a cylinder of height h and radius r is $V = \pi r^2 h$ and its surface area S is $S = 2\pi rh + 2\pi r^2$.)

2. (8 pts. each) Solve each of the following equations for x.

(a) $4^{2x-x^2} = 1$.

(b) $\log_9(4x - 1) = 2$.

(c) $5 = 1 + 4\,e^{-6x}$.

3. (8 pts. each) Compute the first derivative of the following functions.

(a) $f(x) = \ln(2x^3 - 5x)$.

(b) $h(x) = \dfrac{e^{-x^2}}{x^2}$.

(c) $g(t) = t^3 \ln(t^2 + 1)$.

4. (10 pts. each) The population P (in thousands of bacteria) of a certain bacterial culture t days after the culture is started is given by $P(t) = \dfrac{25}{1 + 2\,e^{-t}}$.

(a) What is the initial population of bacteria?

(b) When does the population reach 20 thousand bacteria (that is, when does $P(t) = 20$)?

(c) What is the population of bacteria in the long run, that is, as $t \to \infty$?

5. (10 pts. each) Let $f(x) = e^{3x-x^3}$.

(a) Find all critical points (that is, both x and y coordinates) for $f(x)$.

(b) Find the intervals of increase and decrease for $f(x)$ and identify all critical points you found in part (a) as relative maxima, relative minima, or neither.