

MATH 108 – 25 APRIL 2011– EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given. This exam is counted out of a total of 100 points.

1. (10 pts.) A cylindrical metal can is to have a volume of  $16\pi$  cubic inches. Find the dimensions (height and radius) of such a can that uses the least amount of metal (that is, that has the smallest surface area). (Hint: The volume  $V$  of a cylinder of height  $h$  and radius  $r$  is  $V = \pi r^2 h$  and its surface area  $S$  is  $S = 2\pi r h + 2\pi r^2$ .)

2. (8 pts. each) Solve each of the following equations for  $x$ .

(a)  $5 = 3 \ln(x)$ .

(b)  $3^{2x-1} = 27$ .

(c)  $\frac{25}{1 + 2e^{-x}} = 3$ .

3. (8 pts. each) Compute the first derivative of the following functions.

(a)  $f(x) = (1 + e^x)^{1/3}$ .

(b)  $h(x) = \frac{\ln(x^2)}{x^2}$ .

(c)  $g(t) = t^3 e^{t^2+1}$ .

4. (10 pts. each) Forensic scientists calculate that  $t$  hours after death, the temperature  $T$ , in degrees centigrade, of a dead body is given by  $T(t) = 20 + 17e^{-.05t}$ .

(a) What is the initial temperature of the body?

(b) Suppose police come upon a murdered man and measure his body temperature at  $30^\circ C$ . About how long ago did the murder take place? (That is, find  $t$  such that  $T(t) = 30$ .)

(c) What is the temperature of the body in the long run, that is, what is the temperature as  $t \rightarrow \infty$ ?

5. (10 pts. each) Let  $f(x) = x^2 e^{1-x}$ .

(a) Find all critical points (that is, both  $x$  and  $y$  coordinates) for  $f(x)$ .

(b) Find the intervals of increase and decrease for  $f(x)$  and identify all critical points you found in part (a) as relative maxima, relative minima, or neither.