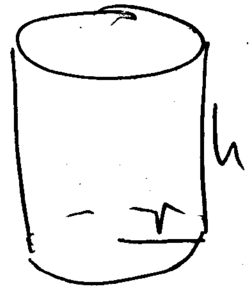
MATH 108 – EXAM 3 – SOLUTIONS

1.



Minimize $S = 2\pi r h + 2\pi r^2$

$$16\cancel{\pi} = \cancel{\pi} r^2 h$$

$$h = \frac{16}{r^2}$$

$$\therefore S = 2\pi r\left(\frac{16}{r^2}\right) + 2\pi r^2$$

$$= \frac{32\pi}{r} + 2\pi r^2$$

$$S' = -\frac{32\pi}{r^2} + 4\pi r$$

$$-\frac{32\pi}{r^2} + 4\pi r = 0$$

$$4\pi r = +\frac{32\pi}{r^2}$$

$$4\pi r^3 = 32\pi$$

$$r^3 = 8$$

$$\underline{r = 2}$$

$$h = \frac{16}{r^2} = \frac{16}{(2)^2}$$

$$= \underline{4}$$

Dimensions are

$r = 2$ in. $h = 4$ in.

2. (a) $5 = 3\ln x$

$$\ln x = \frac{5}{3}$$

$$x = e^{5/3}$$

(b) $3^{2x-1} = 27$

$$3^{2x-1} = 3^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$x = 2$$

(c) $\dfrac{25}{1+2e^{-x}} = 3$

$$25 = 3 + 6e^{-x}$$

$$22 = 6e^{-x}$$

$$e^{-x} = \frac{22}{6}$$

$$-x = \ln(22/6) =$$

$$x = -\ln(22/6) = -\ln(11/3)$$

V1

3. (a) $f(x) = (1+e^x)^{1/3}$

$$f'(x) = \frac{1}{3}(1+e^x)^{-2/3}(e^x)$$

$$= \frac{1}{3}e^x(1+e^x)^{-2/3} \quad //$$

(b) $h(x) = \dfrac{\ln(x^2)}{x^2} = \dfrac{2\ln(x)}{x^2}$

$$h'(x) = \frac{x^2\left(\frac{2}{x}\right) - (2x)(2\ln(x))}{x^4}$$

$$= \frac{2x - 2x(2\ln(x))}{x^4}$$

$$= \frac{2x(1-2\ln(x))}{x^4}$$

$$= \frac{2(1-2\ln(x))}{x^3} \quad //$$

(c) $g(t) = t^3 e^{t^2+1}$

$$g'(t) = t^3(2t\,e^{t^2+1}) + 3t^2 e^{t^2+1}$$

$$= t^2 e^{t^2+1}(2t^2+3) \quad //$$

VI

4. (a) $T(0) = 20 + 17e^{-.05(0)}$

$= 20 + 17 = 37°C$ //

(b) $20 + 17e^{-.05t} = 30$

$17e^{-.05t} = 10$

$e^{-.05t} = \frac{10}{17}$

$-.05t = \ln\left(\frac{10}{17}\right)$

$t = \frac{\ln(10/17)}{-.05} \approx 10.6$ hrs //

(c) $\lim_{t \to \infty} 20 + 17e^{-.05t} = 20 + 17(0)$

$= 20°C$ //

$\checkmark$

5. $f(x) = x^2 e^{1-x}$

(a) $f'(x) = x^2(-e^{1-x}) + 2xe^{1-x}$

$= (2x - x^2)(e^{1-x})$

$(2x - x^2)e^{1-x} = 0$

$x(2-x)e^{1-x} = 0$

$\underline{x=0}$ $\underline{x=2}$ critical numbers

$f(0)=0$ $f(2) = 4e^{-1}$

$(0,0)$ $(2, 4/e)$ critical points //

(b)

$$\begin{array}{c|ccccccccc} & - & - & - & 0 & + & + & + & 0 & - & - & - \\ f' & -1 & & & 0 & & 1 & & 2 & & 3 & \end{array}$$

$f'(-1) = (-1)(3)(+) < 0$
$f'(1) = (1)(2)(+) > 0$
$f'(3) = (3)(-1)(+) < 0$

f decreasing on $(-\infty, 0) \cup (2, \infty)$
increasing on $(0,2)$ //

$(0,0)$ relative min
$(2, 4/e)$ relative max //

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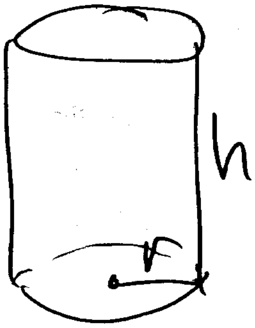

Maximize $V = \pi r^2 h$

$24\pi = 2\pi r h + 2\pi r^2$

$12 = rh + r^2$

$h = \dfrac{12 - r^2}{r}$

$\therefore V = \pi r^2\left(\dfrac{12-r^2}{r}\right) = \pi r(12 - r^2) = 12\pi r - \pi r^3$

$V' = 12\pi - 3\pi r^2$

$12\pi - 3\pi r^2 = 0$

$3\pi r^2 = 12\pi$

$r^2 = 4$

$\underline{r = 2}$ $\quad \cancel{r = -2}$

$h = \dfrac{12 - r^2}{r} = \dfrac{12 - (2)^2}{2}$

$= \dfrac{12-4}{2} = \underline{4}$

Dimensions are

$r = 2$ in $\quad h = 4$ in.

V2

2. (a) $4^{2x-x^2}=1$

$4^{2x-x^2}=4^0$

$2x-x^2=0$

$x(2-x)=0$

$\underline{x=0}$ $\underline{x=2}$ //

(b) $\log_9(4x-1)=2$

$4x-1=9^2$

$4x-1=81$

$4x=82$

$x=20.5$ //

(c) $5=1+4e^{-6x}$

$4=4e^{-6x}$

$1=e^{-6x}$

$-6x=0$

$x=0$ //

V2

3. (a) $f(x) = \ln(2x^3 - 5x)$

$$f'(x) = \frac{6x^2 - 5}{2x^3 - 5x} \quad //$$

(b) $h(x) = \dfrac{e^{-x^2}}{x^2}$

$$h'(x) = \frac{x^2(-2x e^{-x^2}) - (2x) e^{-x^2}}{x^4}$$

$$= \frac{-e^{-x^2}(2x^3 + 2x)}{x^4}$$

$$= \frac{-2\cancel{x} e^{-x^2}(x^2+1)}{x^{\cancel{4}\,3}}$$

$$= \frac{-2e^{-x^2}(x^2+1)}{x^3} \quad //$$

(c) $g(t) = t^3 \ln(t^2+1)$

$$g'(t) = t^3 \cdot \frac{2t}{t^2+1} + 3t^2 \ln(t^2+1)$$

$$= t^2\left(\frac{2t^2}{t^2+1} + 3\ln(t^2+1)\right) //$$

V2

4. (a) $P(0) = \dfrac{25}{1+2e^{-0}} = \dfrac{25}{1+2} = \dfrac{25}{3} \approx 8.3$ thousand

(b) $20 = \dfrac{25}{1+2e^{-t}}$

$20 + 40e^{-t} = 25$

$40e^{-t} = 5$

$e^{-t} = \dfrac{5}{40} = \dfrac{1}{8}$

$-t = \ln(1/8)$

$t = -\ln(1/8) = \ln(8) \approx 2.1$ days

(c) $\lim\limits_{t \to \infty} \dfrac{25}{1+2e^{-t}} = \dfrac{25}{1+2(0)} = 25$ thousand

V2

5. $f(x) = e^{3x-x^3}$

(a) $f'(x) = (3-3x^2)\,e^{3x-x^3}$

$e^{3x-x^3}(3-3x^2) = 0$

$3e^{3x-x^3}(1-x^2) = 0$

$3e^{3x-x^3}(1-x)(1+x) = 0$

$\underline{x=1}$ $\underline{x=-1}$ crit #s.

$f(1) = e^2$ $f(-1) = e^{-2}$

$(1, e^2)$ $(-1, e^{-2})$ critical points //

(b) f': $- - - 0 + + + 0 - - - -$

-2, -1, 0, 1, 2

$f'(-2) = 3(+)(1-4) < 0$

$f'(0) = 3e^0(1) > 0$

$f'(2) = 3(+)(1-4) < 0$

f decreasing on $(-\infty, -1) \cup (1, \infty)$
increasing on $(-1, 1)$ //

$(-1, e^{-2})$ is a relative min
$(1, e^2)$ is a relative max //

V2