MATH 108 – 28 MARCH 2011– EXAM 2

Answer each of the following questions. Show all work, as partial credit may be given.

1. Let $f(x) = \dfrac{x^2}{x+1}$.

   (a) (10 pts.) *Use increments to estimate* how much $f(x)$ will change if x decreases from 2 to 1.8.

   (b) (5 pts.) Find the *exact change* in $f(x)$ if x decreases from 2 to 1.8.

2. (10 pts.) A manufacturer's total cost is $C(q) = 0.01q^3 - 0.05q^2 + 50q + 2000$ dollars when the level of production is q units. The current level of production is 40 units, and the manufacturer is planning to increase this to 41 units. *Use marginal analysis* to estimate the cost of manufacturing the 41st unit.

3. (10 pts.) Find the equation of the tangent line to the curve given by the equation $xy^3 - x^3y = 6$ that passes through the point $(2, -1)$.

4. (10 pts.) Let $f(x) = \dfrac{5x^2}{x^2 + x - 2}$.

   (a) Find all vertical and horizontal asymptotes of $f(x)$.

   (b) Given that $f'(x) = \dfrac{5x(x-4)}{(x^2 + x - 2)^2}$, find the intervals on which $f(x)$ is increasing and decreasing.

   (c) Using the information you found in parts (a), and (b), sketch the graph of $f(x)$. The sketch should accurately reflect the basic shape of the graph.

5. (10 pts. each) Let $f(x) = x^4 - 2x^2 + 2$.

   (a) Find all critical numbers for $f(x)$ and find its intervals of increase and decrease.

   (b) Find all critical *points* of $f(x)$ and identify them as relative maxima, relative minima or neither.

   (c) Find the intervals on which the graph of $f(x)$ is concave up and concave down. Find all inflection *points* of $f(x)$.

   (d) Sketch the graph of $f(x)$, labeling all critical points and inflection points with their coordinates. The sketch should accurately reflect the basic shape of the graph.