MATH 108 - EXAM 2 - VERSION 1 - SOLUTIONS

1. $f(x) = \dfrac{x^2}{x+1}$

(a) $\Delta f \approx f'(x)\Delta x \quad x = 2 \quad \Delta x = -.2$

$$f'(x) = \frac{(x+1)(2x) - (x^2)(1)}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$$

$$f'(2) = \frac{4+4}{(2+1)^2} = \frac{8}{9} \quad \Delta f \approx \left(\frac{8}{9}\right)\left(\frac{-1}{5}\right) = \frac{-8}{45}$$

$$\approx -.17778 \,//$$

(b) $\Delta f = f(1.8) - f(2)$

$$= \frac{(1.8)^3}{(2.8)} - \frac{2^3}{3} = -.17619 \,//$$

2. $C(q) = 0.01q^3 - 0.05q^2 + 50q + 2000$

$$C'(q) = .03q^2 - .10q + 50$$

$$C'(40) = .03(40)^2 - .10(40) + 50$$

$$= 94 \text{ dollars} \,//$$

3. $xy^3 - x^3y = 6 \qquad (2,-1)$

$$\frac{d}{dx}(xy^3 - x^3y) = \frac{d}{dx}(6)$$

$$x \cdot 3y^2 \frac{dy}{dx} + y^3 - x^3\frac{dy}{dx} - 3x^2y = 0$$

Letting $x=2$, $y=-1$ gives

$$6\frac{dy}{dx} - 1 - 8\frac{dy}{dx} + 12 = 0$$

$$\frac{dy}{dx} = \frac{11}{2} \longrightarrow$$ Tangent line:

$$y+1 = \frac{11}{2}(x-2)$$

$$y = \frac{11}{2}x - 12 \quad //$$

4. $f(x) = \dfrac{5x^2}{x^2+x-2}$

(a) Horizontal asymptotes:

$\lim_{x \to \infty} \dfrac{5x^2}{x^2+x-2} = 5$

$y=5$
Horizontal asymptote

Vertical asymptote

$x^2+x-2 = 0$

$(x-1)(x+2) = 0$

$x=1 \quad x=-2$

$x=1$
$x=-2$
Vertical asymptotes

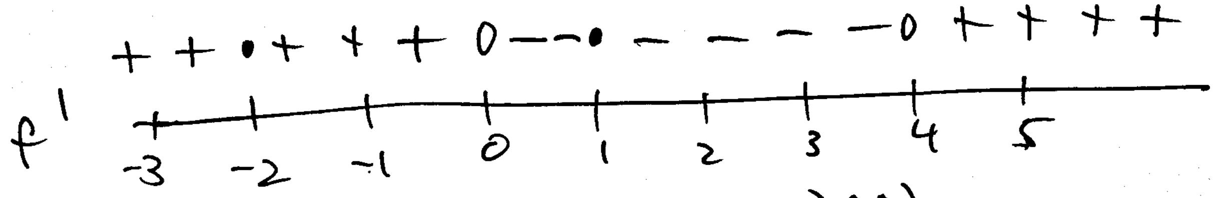
VI

(b) $f'(x) = \dfrac{5x(x-4)}{(x-1)^2(x+2)^2}$

Critical numbers:

$x = 0 \quad x = 4 \leftarrow f' = 0$

$x = 1 \quad x = -2 \leftarrow f'$ undefined



$f'(-3) = \dfrac{(-15)(-7)}{(+)} > 0 \qquad f'(-1) = \dfrac{(-5)(-5)}{(+)} > 0$

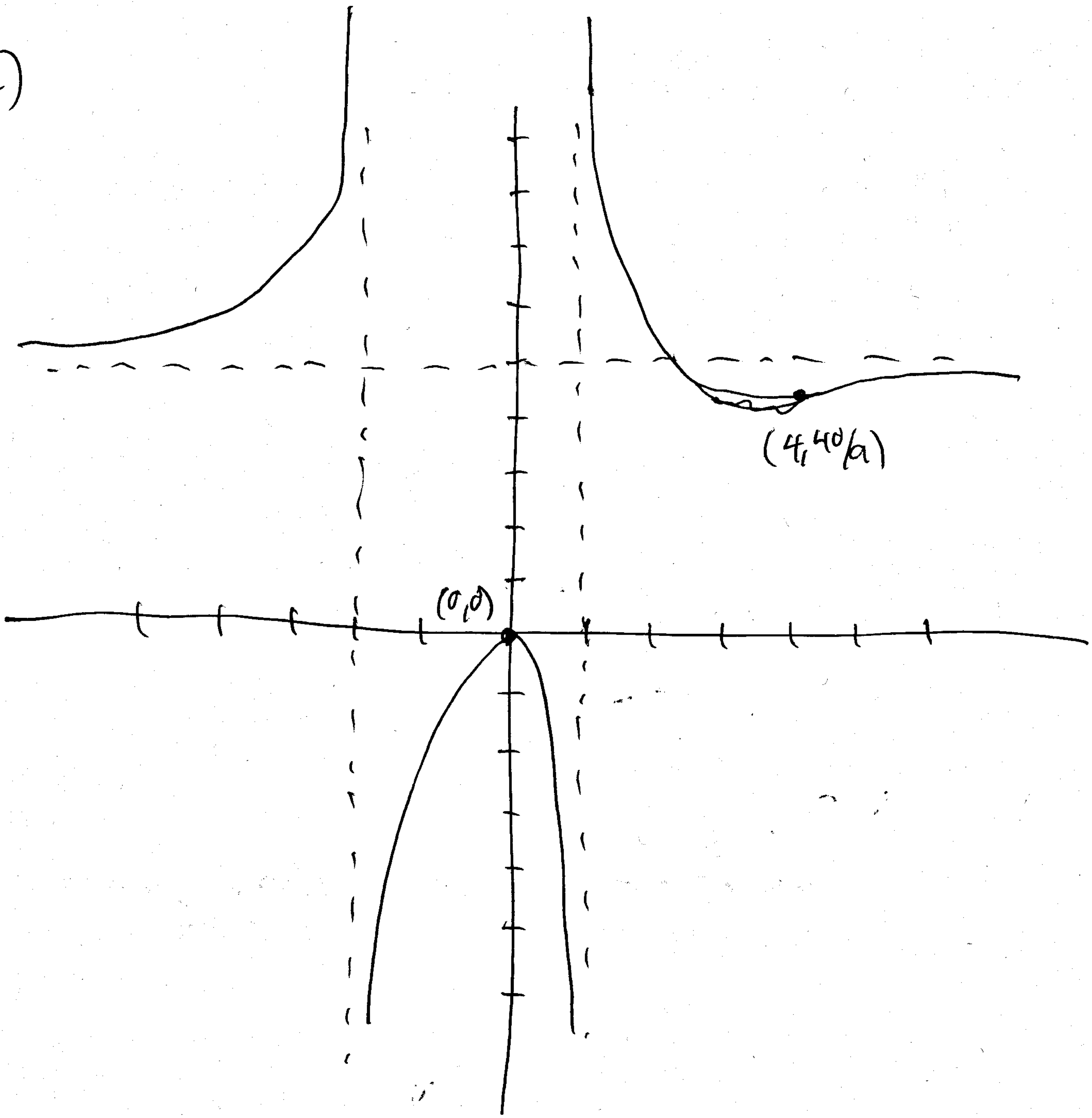
$f'(\tfrac{1}{2}) = \dfrac{(\frac{5}{2})(-\frac{7}{2})}{(+)} < 0 \qquad f'(2) = \dfrac{(10)(-2)}{(+)} < 0$

$f'(5) = \dfrac{(25)(1)}{(+)} > 0$

$\therefore$ f increasing on $(-\infty, -2) \cup (-2, 0) \cup (4, \infty)$

f decreasing on $(0,1) \cup (1,4)$ //

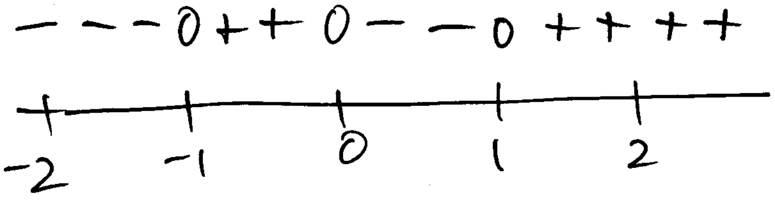
(c)



Critical points: (0,0), (4, 40/9)

5. $f(x) = x^4 - 2x^2 + 2$ vl

(a) $f'(x) = 4x^3 - 4x$



$4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$4x(x+1)(x-1) = 0$

Critical numbers:

$x = 0,\ x = -1,\ x = 1$

$f'(-2) = (-8)(-1)(-3) < 0$

$f'(-\frac{1}{2}) = (-2)(\frac{1}{2})(-\frac{3}{2}) > 0$

$f'(\frac{1}{2}) = (\frac{1}{2})(\frac{3}{2})(-\frac{1}{2}) < 0$

$f'(2) = (8)(3)(1) > 0$

f increasing on $(-1,0) \cup (1,\infty)$

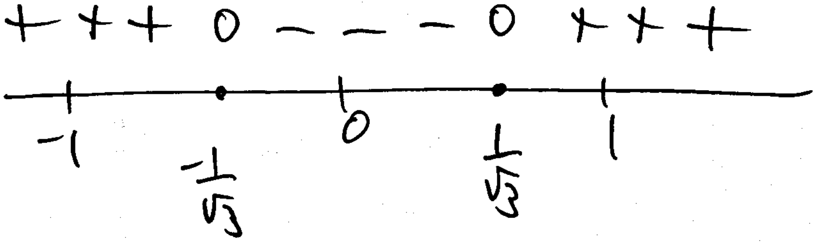
f decreasing on ~~[illegible]~~ $(-\infty,-1) \cup (0,1)$ //

(b) Critical points $(0,2)$ relative maximum

$(-1,1)$ relative minimum

$(1,1)$ relative minimum. //

(c) $f''(x) = 12x^2 - 4$



$12x^2 - 4 = 0$

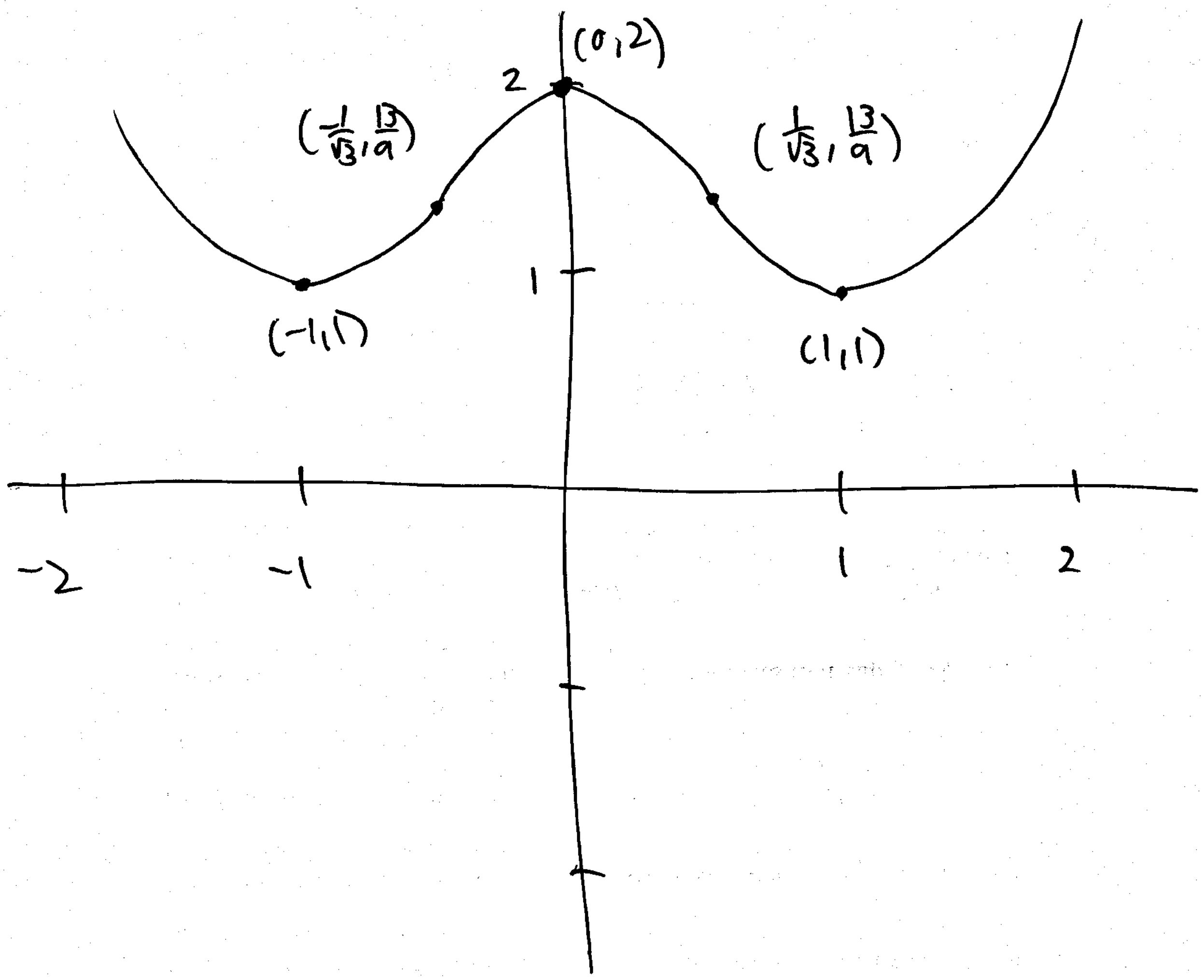
$4(3x^2 - 1) = 0$

$x = \frac{1}{\sqrt{3}},\ x = \frac{-1}{\sqrt{3}}$

$f''(-1) > 0 \quad f''(0) < 0 \quad f''(1) > 0$

f concave up on $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$ conc. down $(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(d)



Inflection points: $(\frac{1}{\sqrt{3}}, \frac{13}{9})$ $(\frac{-1}{\sqrt{3}}, \frac{13}{9})$

VI

MATH 108 - EXAM 2 - VERSION 2 - SOLUTIONS

1. $f(x) = 3x + \frac{2}{x+1}$

(a) $\Delta f \cong f'(x)\Delta x \quad x=2,\ \Delta x = .2$

$f'(x) = 3 - \frac{2}{(x+1)^2} \qquad f'(2) = 3 - \frac{2}{3^2} = \frac{25}{9}$

$\Delta f \approx \left(\frac{\overset{5}{\cancel{25}}}{9}\right)\left(\frac{\overset{1}{\cancel{2}}}{\underset{\cancel{5}}{\cancel{10}}}\right) = \frac{5}{9} \approx .5556$ //

(b) $\Delta f = f(2.2) - f(2)$

$= 3(2.2) + \frac{2}{(2.2+1)} - 3 \cdot 2 - \frac{2}{2+1}$

$= 6.6 + \frac{2}{3.2} - 6 - \frac{2}{3}$

$\approx .55833$ //

2. $Q(t) = .002t^3 + 0.1t + 3.4$

$Q'(t) = .006t^2 + 0.1$

$Q'(10) = .006(100) + 0.1 = 0.7$ parts per million //

3. $x^2y^3 - 2y = 6x - 5 \qquad (1,-1)$

$$\frac{d}{dx}\left(x^2y^3 - 2y\right) = \frac{d}{dx}(6x-5)$$

$$x^2 \cdot 3y^2\frac{dy}{dx} + 2xy^3 - 2\frac{dy}{dx} = 6$$

Letting $x=1, y=-1$ gives

$$3\frac{dy}{dx} - 2 - 2\frac{dy}{dx} = 6$$

$$\therefore \frac{dy}{dx} = 8$$

Tangent line:

$$y+1 = 8(x-1)$$

$$y = 8x - 9 \;//$$

V2

4. $f(t) = \frac{3t^2}{t^2-t-2} = \frac{3t^2}{(t+1)(t-2)}$

(a) Horizontal asymptote

$\lim_{t\to\infty} \frac{3t^2}{t^2-t-2} = 3$ $\therefore$ $y=3$ is the horizontal asymptote //

Vertical asymptote

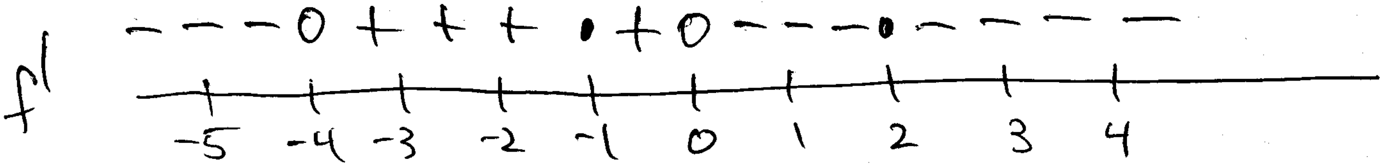
$t^2-t-2=0$

$(t+1)(t-2)=0$

$t=-1,\ t=2$

$\therefore$ $t=-1,\ t=2$ are the vertical asymptote //

(b) $f'(t) = \frac{-3t(t+4)}{(t+1)^2(t-2)^2}$ Critical numbers: $t=0,\ t=-4$, $t=-1,\ t=2$



$f'(-5) = \frac{(15)(-1)}{(+)(+)} < 0$ $f'(-2) = \frac{(6)(2)}{(+)(+)} > 0$

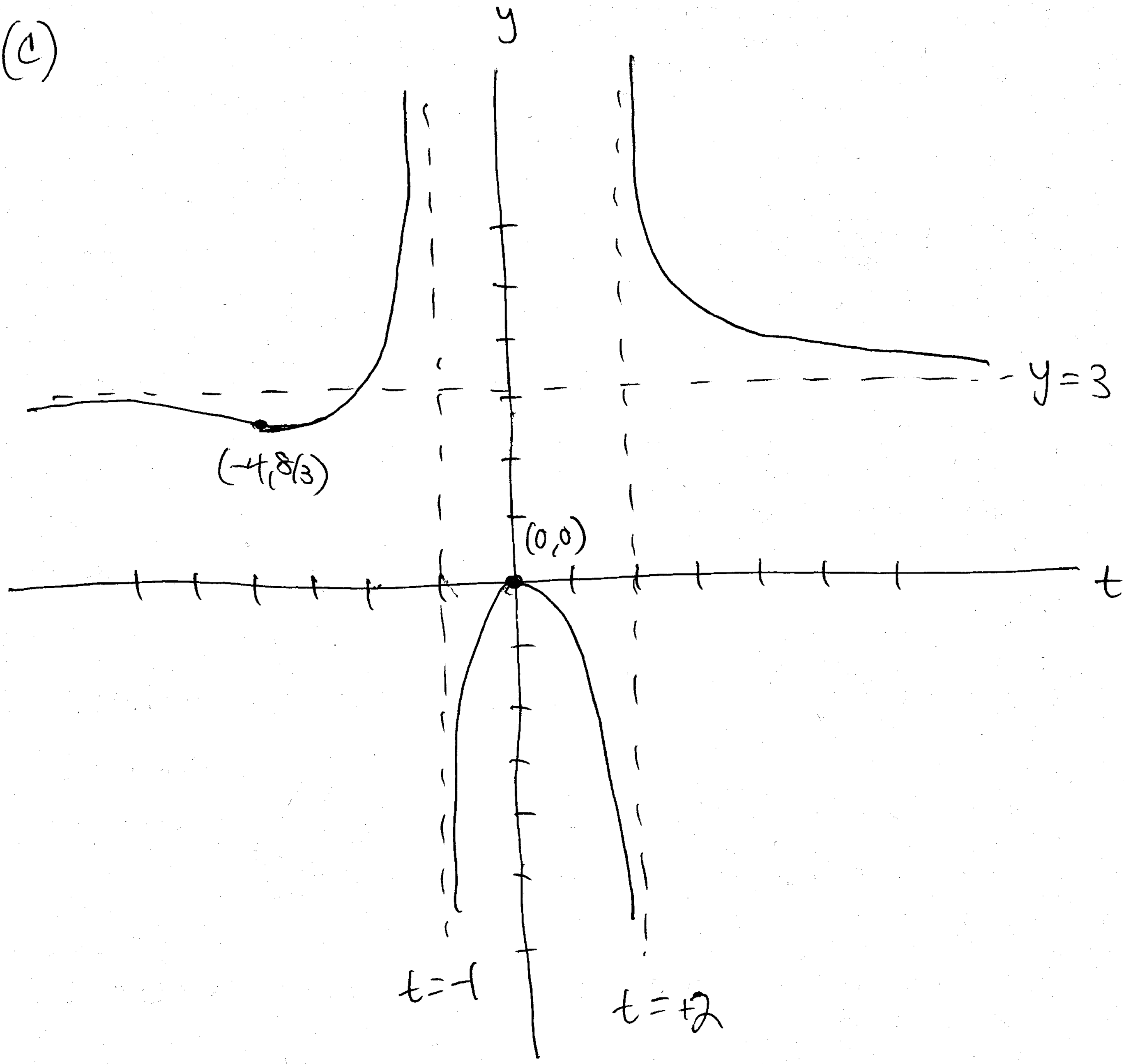
$f'(-\frac{1}{2}) = \frac{(\frac{3}{2})(\frac{7}{2})}{(+)(+)} > 0$ $f'(1) = \frac{(-3)(5)}{(+)(+)} < 0$ $f'(3) = \frac{(-9)(7)}{(+)(+)} < 0$

f increasing $(-4,-1) \cup (-1,0)$

f decreasing $(-\infty,-4) \cup (0,2) \cup (2,\infty)$

V2

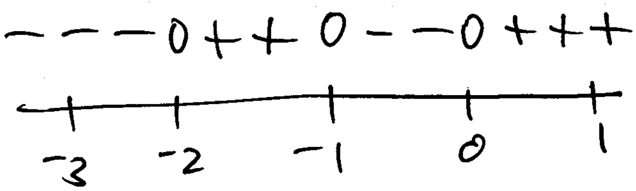
(c)



Critical points: $(-4, 8/3)$, $(0,0)$

5. $f(x) = x^4 + 4x^3 + 4x^2$

(a) $f'(x) = 4x^3 + 12x^2 + 8x$



$4x^3 + 12x^2 + 8x = 0$

$4x(x^2 + 3x + 2) = 0$

$4x(x+1)(x+2) = 0$

$f'(-3) = (-12)(-2)(-1) < 0$

$f'(-\frac{3}{2}) = (-6)(-\frac{1}{2})(\frac{1}{2}) > 0$

$f'(-\frac{1}{2}) = (-2)(\frac{1}{2})(\frac{3}{2}) < 0$

$f'(1) = (4)(2)(3) > 0$

Critical numbers:

$x = 0,\ x = -1,\ x = -2$

$\therefore$ f increasing on $(-2,-1) \cup (0,\infty)$

f decreasing on $(-\infty,-2) \cup (-1,0)$

(b) $f(0) = 0$

$f(-1) = 1$

$f(-2) = 16 - 32 + 16 = 0$

Critical points: (0,0) ~~to~~ relative minimum

(-1,1) relative maximum

(-2,0) relative minimum

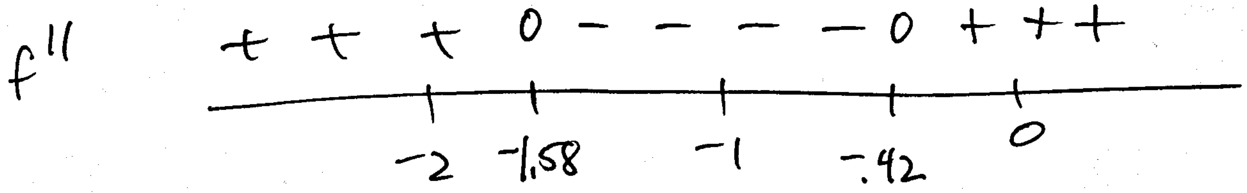
(c) $f''(x) = 12x^2 + 24x + 8$

$12x^2 + 24x + 8 = 0$

$4(3x^2 + 6x + 2) = 0$

$$x = \frac{-6 \pm \sqrt{36-24}}{6} = \frac{-6 \pm \sqrt{12}}{6} = -1 \pm \frac{\sqrt{3}}{3}$$

$x \approx -.42 \qquad x \approx -1.58$



$f''(-2) = 4(12 - 12 + 2) > 0$

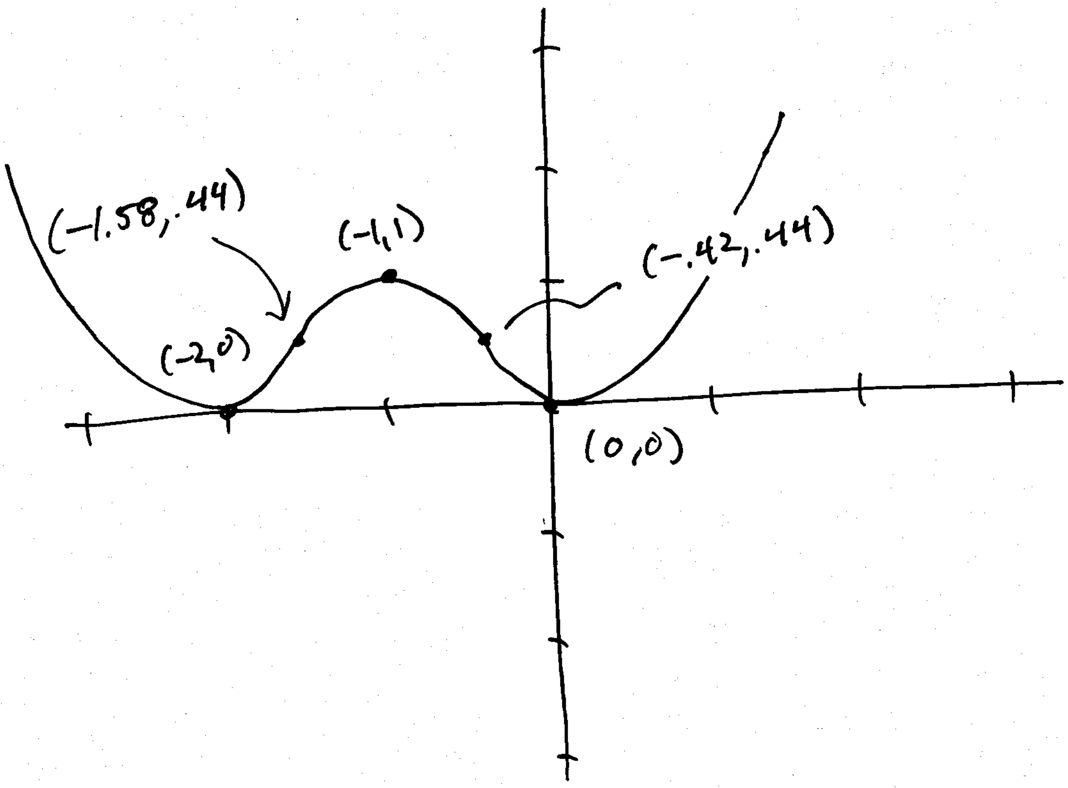
$f''(-1) = 4(3 - 6 + 2) < 0$

$f''(0) = 8 > 0$

f concave up on $(-\infty, -1.58)$ $\cup (-.42\cancel{3}, \infty)$

f conave down on $(-1.58, -.42)$ //

(d)



Inflection points: ~~(1.88)~~ ~~(-1.58, 3.04)~~
(-1.58, .44)
~~(.42)~~ (-.42, .44)