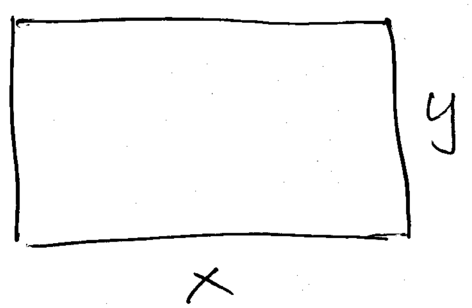
Solutions - Exam 1 - Math 108 (version 1)

1.



P = perimeter of field
x = width of field
y = length of field
A = area of field

$P = 2x + 2y$

$320 = 2x + 2y$

$160 = x + y$

$y = 160 - x$

(or $x = 160 - y$)

$A = xy$

$A = x(160 - x)$ //

(or $A = (160 - y)y$).

2. (a) $\lim\limits_{x \to 1} \dfrac{x^2 + 4x + 5}{x - 1}$ does not exist //
since it evaluates to $\frac{10}{0}$.

(b) $\lim\limits_{x \to \infty} \dfrac{x^2 + 4x + 5}{1 - 2x^2} = \lim\limits_{x \to \infty} \dfrac{x^2}{-2x^2} = -\dfrac{1}{2}$ //

3. (a) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x^2 + x) = 0$ //

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 3x - 2 = -2$ //

(b) $f(x)$ is <u>not</u> continuous at $x = 0$ because $\lim_{x \to 0} f(x)$ does not exist

4. $\lim_{x \to 2^-} \frac{x^2+4}{x-2} = \underline{\underline{-\infty}}$ since if $x < 2$, $x^2+4 > 0$ and $x - 2 < 0$ so $\frac{x^2+4}{x-2} < 0$.

5. $f'(x) = 3x^2$ $f'(-1) = 3$ ← slope of tangent line

$f(-1) = -2$ $(-1, -2)$ ← point on tangent line

$y + 2 = 3(x+1)$

$y = 3(x+1) - 2 = 3x + 1$ //

6. $N(t) = 10t^3 + 5t + t^{1/2}$

$N'(t) = 30t^2 + 5 + \frac{1}{2}t^{-1/2}$

$N'(9) = 30 \cdot 81 + 5 + \frac{1}{2}9^{-1/2}$

$= 2430 + 5 + \frac{1}{6}$

$= 2435\frac{1}{6} \approx 2435 \frac{\text{persons}}{\text{day}}$ //

The infected population is <u>increasing</u> //

7. (a) $f'(x) = 4(2+5x)^3(5) = 20(2+5x)^3$ //

(b) $y' = x^3[3(x-5)^2] + (x-5)^3(3x^2)$

$= 3x^3(x-5)^2 + 3x^2(x-5)^3$ //

$= 3x^2(x-5)^2[x + (x-5)]$

$= 3x^2(x-5)^2(2x-5)$ //

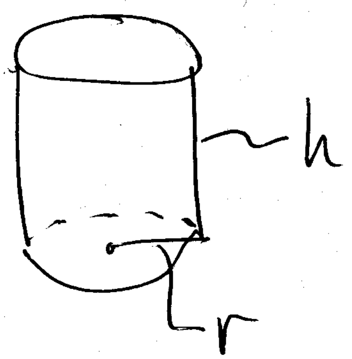
(c) $g(x) = \dfrac{2x}{3x+1}$

$$g'(x) = \frac{(3x+1)(2)-(2x)(3)}{(3x+1)^2}$$

$$= \frac{6x+2-6x}{(3x+1)^2} = \frac{2}{(3x+1)^2} \quad //$$

Solutions – Exam 1 – Math 108
(Version 2)

1.



$$S = 2\pi r h + 2\pi r^2$$

$$\boxed{\begin{aligned} V &= \pi r^2 h \\ 7\pi &= \pi r^2 h \\ 7 &= r^2 h \\ h &= \frac{7}{r^2} \end{aligned}}$$

$$\therefore S = 2\pi r\left(\frac{7}{r^2}\right) + 2\pi r^2$$

$$= \frac{14\pi}{r} + 2\pi r^2 //$$

2. (a) $$\lim_{x\to 3} \frac{9-x^2}{x-3} = \lim_{x\to 3} \frac{\overset{-1}{\cancel{(3-x)}}(3+x)}{\cancel{x-3}}$$

$$= \lim_{x\to 3} -(3+x) = -6 //$$

(b) $$\lim_{x\to\infty} \frac{2x+3}{x^2+1} = \lim_{x\to\infty} \frac{2x}{x^2} = \lim_{x\to\infty} \frac{2}{x} = 0 //$$

6. $N(x) = 6x^3 + 500x + 8000$

$N'(x) = 18x^2 + 500$

$N'(8) = 18 \cdot 64 + 500$

$= 1652 \ \frac{\text{persons}}{\text{week}}$ //

Usage is <u>increasing</u> at $x=8$. //

7. (a) $f'(x) = \frac{1}{2}(2+5x)^{-1/2}(5) = \frac{5}{2}(2+5x)^{-1/2}$

(b) $y' = x^2[5(2x+3)^4(2)] + (2x+3)^5(2x)$

$= 10x^2(2x+3)^4 + 2x(2x+3)^5$ //

$= 2x(2x+3)^4[5x + (2x+3)]$

$= 2x(2x+3)^4(7x+3)$ //

(c) $g(x) = \frac{x}{2-x}$ $\quad g'(x) = \frac{(2-x)(1)-(x)(-1)}{(2-x)^2}$

$= \frac{2-x+x}{(2-x)^2} = \frac{2}{(2-x)^2}$ //

3. (a) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2 - x^3 = 1$ //

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2+1}{x+1} = \frac{2}{2} = 1$ //

(b) $f(x)$ is continuous at $x = 1$

because $f(1) = 2 - 1 = 1 = \lim_{x \to 1} f(x)$.

4. $\lim_{x \to 2^+} \frac{x^2+1}{2x-4} = +\infty$ since if $x > 2$, $x^2+1 > 0$, and $2x - 4 > 0$.

so $\frac{x^2+1}{2x-4} > 0$ if $x > 2$.

5. $f'(x) = -3x^2$ $f'(1) = -3$ ← slope of tangent line

$f(1) = 2 - 1 = 1$ $(1,1)$ ← point on tangent line

$y - 1 = -3(x-1)$ ∴ $y = -3(x-1) + 1 = -3x + 4$ //