5.1. Antidifferentiation: The Indefinite Integral

Antidifferentiation

A function $F(x)$ is said to be an antiderivative of $f(x)$ if

$$F'(x) = f(x)$$

for every x in the domain of $f(x)$. The process of finding antiderivatives is called antidifferentiation or indefinite integration.

Example

Verify that $F(x) = \frac{1}{3}x^3 + 5x + 2$ is an antiderivative of $f(x) = x^2 + 5$.

The Indefinite Integral

Fundamental property of Antiderivatives

If $F(x)$ is an antiderivative of the continuous function $f(x)$, then any other antiderivative of $f(x)$ has the form $F(x) + C$ for some constant C.

The Indefinite Integral

The family of all antiderivatives of $f(x)$ is written

$$\int f(x)\; dx = F(x) + C$$

and is called the indefinite integral of $f(x)$.

The integral symbol is $\int$, the function $f(x)$ is called the integrand, C is the constant of integration, and dx is a differential that indicates x is the variable of integration.

Rules for Integrating Common Functions

- The constant rule: $\int k \, dx = kx + C$ for constant k
- The power rule: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for all $n \neq -1$
- The logarithmic rule: $\int \frac{1}{x} \, dx = \ln|x| + C$ for all $x \neq 0$
- The exponential rule: $\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$ for $k \neq 0$

Example

Find these integrals:

a. $\int x^{15} \, dx$

b. $\int e^{3x} \, dx$

Algebraic Rules for Indefinite Integration

- The constant multiple rule:

$$\int kf(x)\ dx = k \int f(x)\ dx \quad \text{for constant } k$$

- The sum rule:

$$\int [f(x) + g(x)]\ dx = \int f(x)\ dx + \int g(x)\ dx$$

- The difference rule:

$$\int [f(x) - g(x)]\ dx = \int f(x)\ dx - \int g(x)\ dx$$

The Indefinite Integral

Example (#6)

Find the indefinite integral

$$\int 3e^{x}\ dx$$

The Indefinite Integral

Example (#10)

Find the indefinite integral

$$\int \left(\frac{1}{x^2} - \frac{1}{x^3} \right) \, dx$$

The Indefinite Integral

Example (#22)

Find the indefinite integral

$$\int y^3 \left(2y + \frac{1}{y}\right) \, dy$$

The Initial Value Problem

A differential equation is an equation that involves derivatives. An initial value problem is a problem that involves solving a differential equation subject to a specified initial condition.

Example (#34)

Solve the initial value problem:

$$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}} \quad \text{where} \quad y = 5 \quad \text{when} \quad x = 4$$

The Initial Value Problem

Example (#36)

Find the function $f(x)$ whose tangent line has slope $3x^2 + 6x - 2$ for each value of x and whose graph passes through the point $(0, 6)$.