# 4.3. Differentiation of Logarithmic and Exponential Functions

## Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \text{for} \quad x > 0$$

### Example

Differentiate the function $f(x) = x \ln \sqrt{x}$.

# Differentiation of Logarithmic Functions

## The Chain Rule for Logarithmic Functions

If $u(x)$ is a differentiable function of $x$, then

$$\frac{d}{dx}[\ln u(x)] = \frac{u'(x)}{u(x)}$$

## Example

Differentiate the function $f(x) = \ln(x^2 + 1)$.

# Differentiation of Logarithmic Functions

## Example

Differentiate the function $f(x) = \ln(x^3 - 5x + 4)$.

# Differentiation of Logarithmic Functions

## Example

Find an equation for the tangent line to $y = x + \ln x$ at the point where $x = e$.

# Differentiation of Exponential Functions

## The Derivative of the Exponential Function

$$\frac{d}{dx}(e^x) = e^x \quad \text{for every real number} \quad x$$

### Example

Differentiate the function $f(x) = \dfrac{e^x}{x}$.

# Differentiation of Exponential Functions

## The Chain Rule for Exponential Functions

If $u(x)$ is a differentiable function of $x$, then

$$\frac{d}{dx}e^{u(x)} = e^{u(x)}u'(x)$$

## Example

Differentiate the function $f(x) = xe^{2x}$.

# Differentiation of Exponential Functions

### Example

Find the largest and smallest values of the function $F(x) = e^{x^2-2x}$ over the closed interval $0 \leq x \leq 2$.

# Logarithmic Differentiation

Differentiating a function that involves products, quotients, or powers can often be simplified by first taking the logarithm of the function.

Step 1. Take logarithms of both sides of the expression for $f(x)$ and simplify the resulting equation.

Step 2. Use the chain rule to differentiate both sides.

Step 3. Multiply both sides with $f(x)$ to get $f'(x)$.

# Logarithmic Differentiation

## Example

Use logarithmic differentiation to find the derivative of $f(x) = \sqrt[4]{\dfrac{2x+1}{1-3x}}$.

# Logarithmic Differentiation

## Example

Use logarithmic differentiation to find the derivative of
$f(x) = \dfrac{e^{3x}(x^2+5)}{(1-x)^5}$.